Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

June 1948 Vol. 1, No. 6

General Theoretical and Experimental Methods

(See also Rev. 1044)

933. **U. G. Tolstov, "An electric device for the solution of homogeneous and nonhomogeneous ordinary linear differential equations"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, 1947, no. 3, pp. 319–322.

For the recurrent formula

$$a_n Y_{n+k} = b_k - (a_o Y_k + \ldots + a_{n-1} Y_{k+n-1}),\ k = 0,1 \ldots\ldots\ldots\ [1]$$

an electrical network analog is given. Potential dividers are used to produce products of a unit potential by a_i/a_n and by Y_i. A series connection produces Y_{n+k}. A commutator provides a convenient way of replacing Y_i by Y_{i+1}. Means for meeting scale trouble are suggested. Since Equations [1] define the coefficients of the Taylor expansion of the solution of $a_o y + a_1 y' + \ldots + a_n y^{(n)} = f(t)$, with $b_k = f^{(k)}(t_o)$, $y^{(k)}(t_o) = Y_k$, for $k = 0, \ldots, n-1$, the author evidently feels that the title of the paper is sufficiently justified. A. W. Wundheiler, USA

934. **W. Kerkhofs, "Solving systems of simultaneous equations with a great number of unknowns (Résolution de systèmes d'équations simultanées à un grand nombre d'inconnues),"** *Ossature métallique*, Apr. 1947, pp. 187–195.

This paper describes methods of solving a system of linear algebraic equations with many small coefficients. The general procedure involves a series of steps, each equivalent to the solution of two equations in two unknowns, and is basically one of successive approximations. Several cases are distinguished, depending on the number of "large" coefficients. Two numerical examples are worked. W. M. Kincaid, USA

General Dynamics, Kinematics, Friction

(See also Revs. 942, 1061)

935. **C. W. Herweyer, "Safety coefficient against derailment at high speeds in straight and curved tracks (Zekerheidscoëfficiënt tegen ontsporen bij het rijden met hoge snelheid op de rechte baan en in bogen),"** *Ingenieur 's-Gravenhage*, May 21, 1948, vol. 60, pp. V.37–50.

The factor of safety against derailment is defined as the ratio between: (1) the horizontal force on the flange of the leading wheel in contact with the rail which is required to cause the wheel to climb the rail, and (2) the force exerted on the flange while traveling on a straight or curved track. An approximation is given for the calculation of the forces due to yawing or nosing of the vehicle. Also a factor of 1.5 is used to allow for impact between wheel flange and rail. For traveling on a curved track no impact factor is employed, on the ground that there is a gradual transition from straight to curved track, and also that there is no yawing on the curved track. It is shown that the factor of safety decreases with a decrease in the wheel base and with increase in speed. In general, factors of safety are larger on the curved than on the straight track. F. Hymans, USA

936. **E. C. B. Corlett and K. Foulger, "Stability losses on flooding,"** *Trans. Instn. nav. Archit. Lond.*, 1947, vol. 89, pp. 112–127.

This presents a short method for calculating stability of a ship in a damaged condition, leading to a simple formula for the minimum number of bulkheads required. The method is said to have been applied with success to merchant ships of eight types. W. P. Roop, USA

937. **C. Temperley, "Intermeshing noncircular algebraic gears,"** *Engineering*, Jan. 16, 1948, vol. 165, pp. 49–52.

This paper describes a procedure for deriving equations of the pitch lines of noncircular gears, i.e., gears of varying gear ratio. Starting with (1) the function relating the angular displacements of the two gears, (2) the function relating the gear ratio and the angular displacement of one of the gears, or (3) the equation of the pitch line of one of the gears, pitch line equations for both gears are developed. Examples of derivations and a table of formulas are included for various algebraic, logarithmic, and trigonometric relations between the angular displacements of the two gears. Charles E. Crede, USA

938. **N. V. Bootenin, "'Degenerate' dynamic systems considered with the help of the 'jump'-hypothesis"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Jan.-Feb. 1948, vol. 12, pp. 3–22.

The paper deals with the paradox of P. Painlevé [*C. R. Acad. Sci. Paris*, 1895, vol. 121, pp. 112–115; "Leçons sur le frottement," Hermann, Paris, 1895], according to which certain problems of dynamics with standard types of initial conditions and with a high coefficient of friction (according to Coulomb's law) do not admit any solution at all or admit more than one solution.

Among the proposals which were given to avoid this paradox [F. Klein, R. von Mises, G. Hamel, L. Prandtl, *Z. Math. Phys.*, 1909, vol. 5, pp. 58–197] the author chooses that one which admits infinite contact forces with a consequent infinite acceleration and instantaneous stoppage of the motion. Several examples are discussed in detail in the hodograph plane. The word "degenerate" appearing in the itle indicates systems with negligible inertia or elastic force. I. Opatowski, USA

939. **C. H. G. Mills, "A capacitance type torquemeter,"** *J. sci. Instrum.*, May 1948, vol. 25, pp. 151–156.

A torquemeter for the measurement of friction torque in a bearing-testing apparatus subjected to alternating loads is described. A parallel plate capacitor attached to the shaft acts as a

pickup unit. Brushes and slip rings connect the pickup to the circuit which contains the following stages: power supply, amplifier, discriminator, limiter, and oscillator.

The instrument was designed for small dynamic torques of the order of 10 lb-ft peak with an accuracy of $\pm$ 5 per cent, a wide frequency response, and shaft speeds up to 4000 rpm. The stability is adequate for static tests of less than 30 minutes duration.

Harry A. Williams, USA

Gyroscopics, Governors, Servomechanisms

(See Revs. 1037, 1051, 1052)

Vibrations, Balancing

(See also Revs. 935, 1049, 1050, 1051, 1052, 1053)

940. E. Volterra, "Dynamic problems of a beam subject to damping (Problemi dinamici della trave in regime ereditario)," *R. C. Accad. Lincei*, 1947, vol. 2, Jan., pp. 42–47; Feb., pp. 178–180.

By substituting an assumed exponential damping function in the general differential-integral equation for a vibrating beam, and defining specific stress in terms of specific strain and the damping function, the author develops a general procedure for the treatment of vibrating beams. A constant cross section and the other conventional limitations are assumed. The method is applied to a simply supported beam which is: (1) freely oscillating; (2) subjected to a sinusoidal oscillating force; (3) subjected to a concentrated load moving along the beam with constant velocity; and (4) subjected to a pulsating concentrated moving load.

Glenn Murphy, USA

941. M. J. Rosovsky, "Application of integro-differential equations to dynamic problems of the theory of elasticity in case of aftereffect" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1947, vol. 11, pp. 329–338.

Volterra's equations are applied to transverse vibrations of a rod (free and forced), radial oscillations of a spherical shell, and to vibrating plates. The material is assumed to be elastic and to show elastic aftereffects.

Michael Sadowsky, USA

942. C. W. Prohaska, "The vertical vibration of ships," *Shipbuilder*, 1947, vol. 54, Oct., pp. 542–546; Nov., pp. 593–599.

This paper, which is an abstract of a paper presented to the Association Technique Maritime et Aéronautique in June 1947, reports on a comprehensive study of the effects of all factors on the vertical two-noded vibration of ships.

Some 74 symmetrical (fore and aft) mass distributions were analyzed by various methods, and a unique functional relation, presented graphically, was found to exist between the parameter $(k^2 - 8q)$ and the centroidal distance of the half-length mass curve from the midship section. In this parameter k is the radius of gyration and the frequency N is obtainable from $q = 10^{13}I/(N^2\Delta L^3)$, where I is the moment of inertia of the midship section, Δ the ship displacement, and L its length. Analyses of 18 asymmetrical cases indicated that only in extreme cases is it necessary to correct the q obtained for the equivalent symmetrical case. No systematic method is found for evaluating the effect of variable moment of inertia; however, it may be estimated from a table summarizing the calculations for 74 cases.

The second portion of the paper covers the effect of entrained water. Lewis' analysis, based on the conformal transformation of the flow about a cylinder, is extended to other cases including those containing bulbs and shaft bossings. Experimental determination of the entrained water for ten hull shapes was in general agreement with the theory. Studies of the manner in which the virtual mass must be integrated along the length of the hull indicate that Lockwood Taylor's studies on an ellipsoid are applicable to ship hulls. A chart is presented for determining e, the ratio of the virtual mass of the entrained water to the displacement.

The final relation for the frequency per minute (using metric units) is given as

$$N = 10^6 R\sqrt{10I/[\Delta L^3 q(1 + e)]}$$

where R is the product of the correction factors for variable inertia, shearing force, and transverse compression and dilation. As a check on the method the author finds excellent agreement in the predictions of the formula and the frequencies noted by various observers for 15 different ships.

J. M. Robertson, USA

Wave Motion, Impact, Seismology

(See also Revs. 947, 976, 989, 1049)

943. N. Zvolinski, "Propagation of a disturbance from a point impulse in an elastic semispace covered with a layer of fluid" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Feb. 21, 1948, vol. 59, pp. 1081–1084.

The solid homogeneous and isotropic semispace $y>0$ is separated from an ideal liquid by a surface $y = 0$. The initial impulse is axial about the line $x = 0$, $y = y_0>0$, and expansive. The pressure is constant at the fluid surface, the shear stresses are zero at the interface, and the normal stresses are equal there. The three potentials for the fluid flow and the longitudinal and transversal wave motion of the solid are chosen as the unknowns, and the boundary and initial conditions stated.

The author uses his earlier results [*Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, 1947, vol. 56, no. 1] yielding three families of plane waves each depending on a parameter θ, and integrates them with respect to θ in the complex plane to obtain the solutions. These quadratures are replaced by approximate expressions. The same method is stated to work for an initial central rotational impulse.

A. W. Wundheiler, USA

Acoustics

944. J. B. Keller and H. B. Keller, "Reflection and transmission of sound by a spherical shell," *J. acoust. Soc. Amer.*, May 1948, vol. 20, pp. 310–313.

A solution of the problem of reflection and transmission, by a spherical shell, of sound waves from a point source at the center of the shell is obtained. It is shown that with the proper boundary conditions the solution reduces to Rayleigh's solution in one case, and to Fresnel's solution in another. The treatment is straightforward and conventional. By applying the solution, a check is made of an approximate treatment of reflection and transmission from arbitrarily curved thin shells as presented in another paper. Good agreement is obtained. Four special forms of the solution are discussed.

R. G. Wilson, USA

Elasticity Theory

(See also Revs. 941, 952)

945. **P. M. Riz, "Theory of elasticity for large deformations, exceeding the limit of proportionality"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Jan. 11, 1948, vol. 59, no. 2, pp. 223–225.

The author develops further the idea of a previous paper on which he collaborated with I. V. Zvolinski [*Appl. Math. Mech. (Prikl. Mat. Mekh.)*, 1939, vol. 2, no. 4]. Both these papers deal with the second-order theory of elastic and elastoplastic strains in the form of differential equations. The approximation of these equations consists in taking into account only the terms of first and second order in the strain components.

Pointing out the significance of the criteria determining the boundaries between the elastic and elastoplastic zones, the author contents himself with general mathematical considerations, without taking up any definite problems which would allow verification of the theory by experimental measurements.

M. T. Huber, Poland

946. **Y. N. Rabotnov, "Equilibrium of an elastic medium with aftereffect"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Jan.-Feb. 1948, vol. 12, pp. 53–62.

Replacing the elastic constants E and ν by suitable time operators E and ν in the solution of an elasticity problem gives the solution of the same problem for a material endowed with viscosity, creep, and relaxation, provided that operators inverse to E and ν exist. Four distinct hypothetic media are operationally characterized. It is shown that the operator associated with the elastic viscous body

$$\sigma = E\epsilon + \mu(d\epsilon/dt)$$

has no inverse. This does not happen to the elastic relaxer

$$nH(d\epsilon/dt) + E\epsilon = \sigma + n(d\sigma/dt)$$

which thus may be treated by operational methods.

Michael Sadowsky, USA

947. **H. Cabannes, "Application of the Laplace transformation to the study of the dynamics of the string (Application du calcul symbolique à l'étude de la dynamique des fils),"** *C. R. Acad. Sci. Paris*, Apr. 12, 1948, vol. 226, pp. 1165–1168.

The integro-differential equation governing the behavior of a nonuniform elastic string subject to a fairly general force distribution is deduced. For two very special cases the integration can be performed with the aid of the Laplace transform.

George Carrier, USA

Experimental Stress Analysis

(See also Revs. 950, 965, 980)

948. **A. A. Markson and R. S. Williams, "Development of an air-operated force-measuring system,"** *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 271–278.

A force-measuring system is described which is based upon the use of a flexible nonmetallic diaphragm as a measuring element. The force applied to the diaphragm on one side is balanced by air pressure on the opposite side. The balancing air pressure is measured and used as a measure of the applied force. The problems associated with the development of the system are described, with details of the mechanical features and operating experience.

R. L. Bisplinghoff, USA

Rods, Beams, Shafts, Spring, Cables, etc.

(See also Revs. 940, 941, 947, 960, 961, 962, 963, 968, 969)

949. **R. Cajar, "Contribution to the calculation of continuous girders with unequal spans (Beitrag zur Berechnung durchlaufender Träger mit ungleichen Feldweiten),"** *Bauplan. Bautech.*, Feb. 1948, vol. 2, pp. 51–52.

The author describes a systematic procedure for determining the moments at the supports in continuous beams with no settlement of the supports. It is based on the Theorem of Three Moments and is applied to 3, 4, and 5-span beams.

Glenn Murphy, USA

950. **M. C. Shaw, C. D. Strang, and O. W. Hart, "Measurement of piston-ring radial-pressure distribution,"** *Soc. auto. Engrs. quart. Trans.*, Jan. 1948, vol. 2, pp. 169–189.

This paper deals with experimental determination of the radial-pressure distribution of piston rings, the knowledge of which is of great value in comparative measurements of engine performance, oil consumption, etc. Consistent radial-pressure pattern provides also a criterion of quality.

The authors discuss the difficulties involved in accurately measuring the radial-pressure distribution and describe a radial-pressure gage developed by NACA which is compensated for the displacements which accompany load measurements.

This gage consists of a guide, locating the piston ring at approximate cylinder diameter, and of 12 equally spaced force-measuring units, with micrometer adjustment and heat-treated beryllium copper-spring dynamometer. The force is measured by strain gages cemented to these dynamometers using a Wheatstone bridge with self-balancing potentiometer.

By radially advancing the measuring units the ring forces are transferred from the guide to these units. Exact cylinder diameter is secured by a master disk and a system of neon tubes which are just aglow when all units are correctly adjusted. Another system of neon tubes indicates lack of contact between the ring and the guide. The accuracy of load measurement is within 0.02 lb, corresponding to a radial displacement of 0.00002 in.

The experimental results, excepting measurements near the gap, are in fair agreement with the mathematical analysis carried out in the appendixes for the piston rings as continuous beams. It is shown that out-of-roundness of a cylinder by 0.005 in. will materially affect the pressure distribution.

Nicholas Sag, Australia

Plates, Disks, Shells, Membranes

(See also Revs. 941, 962)

951. **N. J. Hoff, B. A. Boley, and L. R. Viggiano, "Stresses in and general instability of monocoque cylinders with cutouts. IV—Pure bending tests of cylinders with side cutout,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1264, Feb. 1948, pp. 1–91.

This paper gives a detailed account of a comprehensive series of experiments in which nine cylinders, each reinforced with stringers and rings, and each of 20-in. diam, were tested in pure bending. On one side of each cylinder there was a cutout bisected by the neutral plane of bending, and therefore lying partly on the tension and partly on the compression side of the cylinder. The angles of the cutouts were 45, 90, or 135 deg; they extended in

seven of the cylinders over about one third of the length of the cylinder, but in the other two over much smaller fractions.

A diagram shows the effect of size of cutout on the critical bending moment, and here there is a marked difference between the behavior of the three cylinders with sixteen stringers and the six cylinders with eight. In the former an increase in the angle of the cutout from 45 to 135 deg had a considerable effect; for comparison the diagram also shows the critical moment (found in earlier experiments) of a cylinder slit along a generator, and so having in effect a cutout of 0 deg. Increase of cutout angle from 0 to 135 deg decreased the critical moment by about one third, but the same increase in the cylinders with eight stringers gave a decrease of less than one sixth.

Some additional results show, in the cylinders with eight stringers, the effect of change in the proportional length of a cutout of 90 deg. Decrease in the proportional length (from the normal one third to about one seventh), in general, increases the critical moment, but there was one exception in which the shortest cylinder used in the series, 39 in. long with one third of its length cut out, gave the highest critical moment of all the cylinders with eight stringers. It was found that the cylinders failed by general instability, defined as simultaneous buckling of stringers and rings together with the sheet attached to them.

There are very full strain-gage measurements and many clear photographs of buckled forms. W. R. Dean, England

952. G. Supino, "On the theory of elastic plates (Sopra la teoria delle lastre elastiche)," *Ann. Mat. pura appl.*, 1945 (issued in 1947), ser. 4, vol. 24, pp. 39–64.

The author considers two types of solutions of the elasticity equations for the case of thick circular plates with loads on the edges only: (1) The Love-Michell solutions satisfying the two boundary conditions of Kirchhoff; (2) the Almansi solution, satisfying the three boundary conditions of Poisson, and producing stresses vanishing towards the center of the plate. Superposing two of these solutions with opposite values of the stress resultants on the generators of the peripheral surface of the plate, the author builds up possible nonzero states of stress with zero resultants on the generators.

This may seem to contradict the de Saint-Venant principle, and the author appears to solve the contradiction as follows: While a stress of zero resultant over a small surface element generates first-order stresses inside, the latter may produce a finite stress when summed for all the elements of the side surface. The author argues, however, that for pure bending problems resultants along generators may be substituted for the stresses proper. It is suggested in the paper that thick plate problems be solved by superposition of solutions of types (1) and (2), which allows the three conditions of Poisson to be satisfied. M. G. Salvadori, USA

953. I. N. Vekua, "On the theory of thin shallow elastic shells" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1948, vol. 12, pp. 69–74.

The author bases his method on V. Vlasov's theory of thin elastic shells. The system of differential equations of that theory is transformed by the author into one equation for the following complex function V

$$V = w + \frac{i\sqrt{12\,(1 - \nu^2)}}{E\delta^2}\,\Phi$$

where w denotes normal displacements of points of the internal shell surface, E Young's modulus, ν Poisson's ratio, δ the thickness of the shell, and Φ a function determining the forces and the moments of the shell. This equation is transformed by the author into a type of Volterra integral equation of a complex domain and its solution is to be found by iteration. Then the author applies his method of determining the function V to shallow spherical shells and to circular cylindrical shells.

Witold Wierzbicki, Poland

954. S. A. Ambartsoomyan, "On the theory of anisotropic shallow shells" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1948, vol. 12, pp. 75–80.

The author deals with an anisotropic thin-walled shell which at every point possesses a plane of elastic symmetry parallel to the inner surface of the shell. The problem is studied in the orthogonal system of curvilinear co-ordinates formed by the curvature lines. After introducing certain simplifying premises and using the general equilibrium equations of shells, as well as V. Vlasov's theory of isotropic shells, the author obtains two equations for the equilibrium of an anisotropic shell. They are expressed in terms of w and φ, where w denotes displacements normal to the surface of the shell and φ a function similar to the Airy function. These two equations are reduced by the author to one equation of the eighth order. The author generalizes Vlasov's formulas for the stability and vibrations of isotropic shells to anisotropic shells. Witold Wierzbicki, Poland

955. Osvaldo Zanaboni, "The principle of reciprocity of shearing stresses in shells and its immediate consequences (Il principio di reciprocità delle tensioni tangenziali nelle lastre a doppia curvatura, e le sue immediate conseguenze)," *Ann. Mat. pura appl.*, 1946 (issued in 1948), ser. 4, vol. 25, pp. 287–311.

In the elastic theory of shells it is customary to make certain simplifying assumptions, one of which is that tangential forces in the plane of the shell are equal in surfaces perpendicular to each other; this usually leads to inconsistencies which are technically not very serious. The author undertakes to derive exact equations, but ignores the derivations in Love's "Theory of elasticity," where the same question is analyzed with the same results.

J. L. Maulbetsch, USA

956. C. Truesdell, "On Sokolovski's 'momentless shells,'" *Trans. Amer. math. Soc.*, Jan. 1947, vol. 61, pp. 128–133.

With reference to other authors who have occupied themselves with the theory of momentless shells (V. V. Sokolovski, P. Neményi), and with reference to an earlier paper of the author himself, large classes of surfaces of revolution are described for which the differential equations of the membrane theory for shells can be solved with the aid of known and tabulated functions (hypergeometric function; Whittaker function). The paper is essentially of a mathematical nature. C. B. Biezeno, Holland

957. K. M. Mooshtari, "The invariant equilibrium equations, in complex form, of the boundary zone of an elastic shell" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Mar.-Apr. 1948, vol. 12, pp. 129–136.

The author follows the customary procedure [see, for example, W. Z. Chien, *Quart. appl. Math.*, 1944, vol. 1, p. 297] to formulate a system of six invariant macroscopic equations of the shell theory. An assumption of the Kirchhoff-Love hypothesis and a definition of the complex stress tensor in terms of the usual macroscopic stress tensor and the quantities depending on the geometry of the middle surface, enable the author to write a set of approximate equilibrium equations for the complex stress tensor. The quality of approximation in these equations depends on the thick-

ness of the shell. It is shown that the complex stress tensor may be obtained from the stress function satisfying a certain fourth-order partial differential equation.

The simplifications resulting from an assumption that the surface of the shell is developable are indicated in some detail.

I. S. Sokolnikoff, USA

958. W. Z. Chien, "Large deflection of a circular clamped plate under uniform pressure," *Chin. J. Phys.*, Dec. 1947, vol. 7, pp. 102–113.

The author presents a solution of the problem by successive approximations through ascending powers of a parameter which represents the ratio of the center deflection of the plate to its thickness. This substitution reduces the original nonlinear differential equations to a sequence of linear differential equations, from which the variable coefficients of the parameter series can be determined.

The results obtained in this manner are in close agreement with those derived by a number of other methods which have been proposed and used for this problem in the past. The chief advantage of the author's approach appears to be that it requires less numerical calculation than the exact power-series solution for the same problem used by S. Way, and that its convergence remains good even for large ratios of the deflection to the thickness of the plate.

The author derives explicit expressions for the stress components in the plate and establishes the condition of yielding along the clamped edge. This calculation is found to be in good agreement with experimental observations by McPherson, Ramberg, and Levy.

M. Hetényi, USA

Buckling Problems

(**See also Rev. 951**)

959. C. B. Biezeno and J. J. Koch, "Note on the buckling of a vertically submerged tube" (in English), *Appl. sci. Res. Sec. A*, 1948, vol. 1, no. 2, pp. 131–138.

A proof is given that a vertically submerged tube will not buckle under its own weight. Three cases are considered: (1) a closed tube, wholly submerged and vertically supported at its bottom; (2) a floating closed tube; (3) a floating tube partially filled with fluid.

The proof consists of the solution of the problem of a submerged tube whose weight is exactly balanced by the water it displaces. This demonstrates that buckling is possible only with extremely small wall thickness and enormous length. The authors thus conclude that the submerged tube solution is practically identical with that for a column of reduced weight over the length buoyed up by the water.

The authors were apparently unaware of the paper "Buckling under locally hydrostatic pressure," by G. H. Handelman [*J. appl. Mech.*, Sept. 1946, vol. 13, p. 198], which discusses end conditions in more detail but does not include the effect of column weight.

D. C. Drucker, USA

960. B. Budiansky, P. Seide, and R. A. Weinberger, "The buckling of a column on equally spaced deflectional and rotational springs," *Nat. adv. Comm. Aero. tech. Note*, no. 1519, Mar. 1948, pp. 1–42.

In this paper the buckling of a continuous column supported by equally spaced deflectional and rotational springs of equal stiffness is investigated by means of the Rayleigh-Ritz energy method. Charts are drawn which give the critical load for columns of two, three, four, and an infinite number of spans in terms of the deflectional and rotational spring parameters. This paper thus extends previous studies of a column on deflectional springs without rotational restraints.

John E. Goldberg, USA

**961. W. Bogucki, "Design of steel columns (Projektowanie słupów stalowych)," ** *Inżyn. Budown.*, Mar. 1948, vol. 5, pp. 116–120.

A formula for the critical stress of a steel beam is derived in terms of its fineness ratio, allowing for eccentricity of the load and nonhomogeneity of the beam.

A. W. Wundheiler, USA

962. B. Budiansky, R. W. Connor, and M. Stein, "Buckling in shear of continuous flat plates," *Nat. adv. Comm. Aero. tech. Note*, no. 1565, Apr. 1948, pp. 1–24.

To supply information for the design of thin-web spars, the theoretical values of shear buckling stress are given for two cases: (1) An infinitely long, clamped plate divided into square panels by intermediate supports; and (2) a plate of infinite length *and* width having a network of intermediate supports which form an array of square panels. It is assumed that the intermediate supports are of infinite bending rigidity but offer no torsional restraint.

The results indicate that, in the first case, a change of the boundary conditions along the transverse edges from simple support to a condition of *continuity*, and, in the second case, a change to continuity of the panels at all four edges provides about 25 and 33 per cent respectively of the increase in shear buckling stress which would be obtained by changing the boundary conditions from simple support to *clamped* edges.

C. T. Wang, USA

963. A. J. S. Pippard, "The critical load of a battened column," *Phil. Mag.*, Jan. 1948, vol. 39, pp. 58–66.

A theoretical treatment is given which is based on a device which has yielded satisfactory results in other problems. The effects of the battens are compared to those of a hypothetical continuous web. The critical buckling load is obtained from the conditions of neutral equilibrium and the condition that the slopes of the deflected column at the battens are equal to those of the hypothetical column.

Marshall Holt, USA

Joints and Joining Methods

(**See also Rev. 982**)

964. R. Alexandre, "The use of arc welding for economic building construction (L'emploi de la soudure à l'arc dans la construction économique des bâtiments)," *Ossature métallique*, Mar. 1948, vol. 13, pp. 139–151.

This article advocates the use of arc welding in the construction of steel frameworks for buildings. Examples are given of the detail design of joints and of the indeterminate calculations arising in such frameworks.

W. S. Hemp, England

965. W. Soete, R. Vancrombrugge, and Ch. De Wulf, "Determination of residual stresses in some welded structures (Détermination des tensions résiduelles dans quelques constructions soudées)," *Ossature métallique*, May 1948, vol. 13, pp. 246–255.

A method for measuring residual stresses using resistance-wire strain gages is presented. A rosette of three gages is placed so that the gage axes intersect at the point at which measurement is

to be made. Zero readings are taken, so that strain measurements may be made as a small hole is drilled in the metal at the point in question. From the change in strain as the drilling progresses the residual stresses, including their variation in depth, may be computed. Using this method a number of welds in tanks and plates were investigated. Welds were made by arc and gas methods, straight and intersecting, and in various welding positions.

The following conclusions have been drawn from the experimental investigation: (1) For straight welds, arc welding gives practically uniaxial stress while gas welding gives more nearly a two-to-one biaxial stress. The magnitude of the stress resulting from arc welding is somewhat greater than for gas welding. (2) At an intersection of two welds at right angles, both arc and gas welding give a nearly balanced biaxial stress state with the greater of the two stresses oriented approximately in the direction of whichever weld was made second. Again gas welding gave slightly lower stresses. (3) The residual stresses increase with depth, the gradient being very steep in some instances. (4) The position in which the welding was performed does not appear to affect either the magnitude or orientation of the residual stresses. Morton B. Millenson, USA

966. J. J. Bikerman, "The fundamentals of tackiness and adhesion," *J. Colloid Sci.*, Feb. 1947, vol. 2, pp. 163–175.

A general discussion of the nature of the forces exerted by liquid and solid adhesives is presented. In the case of a liquid adhesive the tensile force exerted by the joint is time-dependent and is governed by the viscous resistance of the liquid to an inward flow produced by the separation of the joint. An illustrative derivation of the formula describing viscous adhesion, attributed to J. Stefan, is given.

Setting brings about an increase of the viscosity coefficient, and eventually Stefan's formula leads to tensile forces greater than the observed breaking strength of the adhesive. In this condition failure is likened to the ordinary fracture of a solid. It is observed that thin layers of adhesive are stronger than thick ones and that the tensile strength of thin layers is noticeably greater than that of the adhesive in bulk form. This difference is attributed to the increased probability of occurrence of decisive flaws in the larger specimens. Robert T. Jones, USA

967. G. Nikolayev and N. Prohorov, "Stresses in structural members during welding" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci.* (*Izv. Akad. Nauk SSSR Ser. tekh. Nauk*), 1947, no. 3, pp. 307–318.

While most previous work has been concerned with the residual stress produced by welding, the present work deals with strain and stress developed during welding. To measure this a layer of arc-weld metal was deposited along one edge of a $^3/_8$-in. mild-steel plate, and the change of temperature and the strain e were recorded near the weld metal as a function of time. If e_t is the thermal expansion, then $e_a = e - e_t$ is the sum of elastic and plastic strains developed during welding at any time. The value of e was measured by Huggenberger extensometers with legs protected from heat by water-cooled fixtures, and e_t was found from the free dilation of a strip of metal heated to the same temperature. The variables studied were: (1) width of plate, from about $^3/_4$ to 8 in.; (2) two-layer welding; (3) water quenching during welding; and (4) alloying element in steel.

The results may be summarized as: (1) The residual stress was below the yield point only for plates less than $2^1/_2$ in. wide. (2) The higher the temperature of the first layer when the second one was started, the smaller were the elastic and plastic strains during subsequent cooling. (3) A substantial decrease of residual stress can be achieved by directing a stream of cold water at the weld while the latter is in the temperature range of 400 to 660 F. (4) The austenite decomposition of a nickel-alloy steel between 930 and 660 F causes a change in the magnitude of strains, in the sense that before the decomposition the trend was toward decreasing, and after decomposition toward increasing this magnitude. This would explain the occurrence of cracks well below 660 F (apparently at 400 F). D. Rosenthal, USA

Structures

(See also Revs. 936, 949, 964, 1014)

968. L. Ross Levin, "Strength of thin-web beams with transverse load applied at an intermediate upright," *Nat. adv. Comm. Aero. tech. Note*, no. 1544, Feb. 1948, pp. 1–20.

It is assumed that a portion of the load applied to the end of an upright is carried by bending of the beam flange on the loaded side, which is assumed for this calculation to be fixed at the adjacent uprights. The remainder of the load is carried by the upright, together with effective area of adjacent web, and is assumed to vary linearly to zero at the unloaded end. On this basis a simple expression for the load in the upright is obtained, to which the compression due to the tension field must be added. The analysis is shown to be in fair agreement with test data.
John E. Goldberg, USA

969. G. Vedeler, "The distribution of load in longitudinal strength calculations," *Trans. Instn. nav. Archit. Lond.*, 1947, vol. 89, pp. 16–31.

The structure of a ship, consisting as it does of a load-carrying shell supported by a number of bulkheads and a gridwork of beams and longitudinals, generally has been analyzed as a unit without much attention to the component parts of the structure. Some progress has been made by H. A. Schade ["Bending theory of ship bottom structure," *Trans. Soc. nav. Arch. mar. Engrs.*, 1938] who treated the bottom structure as an anisotropic plate, and by the methods of beams on elastic foundations, as discussed by M. Hetényi ["Beams on elastic foundations," University of Michigan Press, 1936].

The author has given the problem comprehensive study from both points of view, although Schade appears to have more closely approached some of the boundary conditions of the bottom structure, and has expounded his results in a book ["Grillage beams in ships and similar structures," Grondahl and Son, Oslo, 1945]. The present paper covers special applications of these methods of analysis in the determination of the transfer of the bottom and deck loads to the longitudinal bulkheads and side plating of a tanker and of a cargo ship. F. E. Reed, USA

970. Louis Baes, "The steel sheet piling 'Belval P' for cellular construction (Les palplanches plates 'Belval P' pour constructions cellulaires)," *Ossature métallique*, Feb. 1948, vol. 13, pp. 75–105.

The properties of this new type of steel sheet piling are investigated. Discussion of the service conditions of this form of construction in civil engineering work is followed by a photoelastic examination on a full-size model, representing a section through the hooked joint. The technique used is very fully described and the surface stresses are deduced in terms of the applied load.

This is followed by a description of some mechanical tests on slices cut from actual rolled-steel sections. Combination of the results of the mechanical tests with the stresses measured photo-

elastically enables the load which can be withstood without appreciable yielding at the hooks to be estimated. Calculations are also made of the deflections and elongations to be expected in the central portion of the section under practical conditions.

It is concluded that the new form of sheet piling made from the selected grade of steel provides good strength characteristics combined with minimum weight and that it has considerable energy-absorbing capacity before fracture.

A. F. C. Brown, England

971. **F. J. Plantema and J. H. Rondeel, "Compression tests on tubes with and without annealed ends"** (in English), *Nat. LuchtLab. Amsterdam Rap.*, no. S.330, Jan. 1948, pp. 1–28.

The authors' tests show that it is satisfactory to "cut off" the column buckling curve at a value equal to the local buckling stress for the annealed material, except for columns with a high degree of end restraint. In such cases the annealing tends to reduce the resistance somewhat below the column curve value, as the local buckling stress is approached. F. R. Shanley, USA

972. **Sven Törnmarck, "Calculation of shock absorbers for landing gears (Beräkning av stötdämpare för landningsställ),"** *Tekn. Skr.*, no. 133, 1947, pp. 1–16.

The paper gives a calculation method for determining the dimensions of an oleopneumatic shock absorber with variable orifice. Dividing the shock absorption between the pneumatic tire and the oleo strut, the different dimensions are calculated by means of curves, formulas, and a nomogram for the orifice. A numerical example is given. P. N. Brandt-Møller, Denmark

973. **W. Muckle, "The design of light-alloy ships' structures,"** *Shipbuilder*, Jan. 1948, vol. 55, pp. 17–27.

The paper draws attention to the possible savings in weight which might be achieved by using aluminum alloys of medium strength instead of steel for ships' structures. Methods of similitude are employed to estimate the changes in dimensions and savings in weight. The effect of a uniform change in temperature or of a linear temperature gradient in a composite structure is examined. The paper shows by simple assumptions that the light alloy structure should be superior to the steel structure in resistance to impact.

The reviewer notes that quite different results would probably have been obtained with different but equally plausible assumptions. H. A. Wills, Australia

974. **J. L. Adam, "Classification societies and the efficiency of hull structures,"** *Trans. N. E. Coast Instn. Engrs. Shipb.*, Feb. 1948, vol. 64, pp. 139–150.

In connection with classification requirements the author gives a survey of points of view to be considered when constructing certain structural members of ships. Some examples illustrate the text. W. L. Esmeijer, Holland

Plastic Flow, Failure; Mechanics of Solid State

(**See also Revs. 941, 945, 946, 965, 966**)

975. **K. H. Swainger, "Stress-strain compatibility in greatly deformed engineering metals,"** *Phil. Mag.*, June 1947, vol. 38, pp. 422–439.

This paper develops stress-strain relations for arbitrarily large deformations of an anisotropic strain-hardening material which obeys the von Mises yield condition. Stress and strain are defined with respect to the deformed state. It is assumed that a normal stress on the surface of an element in the deformed state produces only normal strains along and across the stress direction. Normal strain in any direction is obtained by superposition of the effects of the normal stress along and across this direction. (This superposition is open to serious objection in the plastic range.)

In place of the usual shear strains, normal strains in the 45-deg planes, and due solely to shear stresses, are introduced. (To compute ordinary shearing strains using these normal strains is awkward even for infinitesimal strains; also, these normal strains cannot be directly observed, as a rule.) The elastic strain is given by Hooke's law and involves the values of Young's modulus and Poisson's ratio in the directions of orthogonal curvilinear fixed co-ordinate lines at the instantaneous position of the element.

Incremental plastic strain is found by substracting incremental elastic strain from incremental total strain which is given in terms of the stress increments by a Hooke's law with variable moduli. Total plastic strain is defined by integration with respect to stress components. In general, this integral will not be linear in the stresses, even though the stress-strain relations obtained are linear.

For the isotropic case, and the plastic moduli constant throughout the deformation, linear relations between stress and strain are obtained in closed form. These constitute a deformation theory based on an assumption of superposition of stress effects in the plastic range, and as such will disagree with the results of general tests with combined stresses. The restriction to loading throughout is not made, but appears to be essential.

H. J. Greenberg, USA

976. **K. A. Rakhmatoolin, "On propagation of cylindrical waves under plastic deformations (torsional impact)"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Jan.-Feb. 1948, vol. 12, pp. 39–46.

This is a theoretical study of the propagation of stress and strain in an elastic-plastic space under instantaneous application of a torsional moment to a rigid, infinitely long cylinder built into the space. Two cases of variation of the applied moment are considered: (1) The moment increases with time from its initial value. For this case it is concluded that an elastic disturbance is propagated outward, followed by a plastic disturbance. It is also concluded that the radius reached by the latter depends only on the initial moment, not on its rate of increase. (2) The applied moment decreases with time from its initial value. For this case the variation of strain in the neighborhood of the applied moment is discussed. M. P. White, USA

(EDITOR'S NOTE: A 12-page translation of this paper has been made by Dr. A. Hrennikoff; the editor can supply photostats at the usual rate.)

977. **C. W. MacGregor, L. F. Coffin, Jr., and J. C. Fisher, "Partially plastic thick-walled tubes,"** *J. Franklin Inst.*, Feb. 1948, vol. 245, pp. 135–158.

The authors analyze partial plastic yielding in thick-walled cylindrical tubes stressed by any combination of internal or external pressure and axial loading. Particular attention is given to the cases in which an elastic and a plastic region may be sharply distinguished. In the elastic region the authors use Lamé's equations. In the plastic region they assume that the sum of the total strains is proportional to the mean of the three principal stresses, which is also in accordance with elasticity theory. They assume

strain hardening, so that the invariant of stress is a given function of the invariant of strain. They also assume small strains, the total strains consisting of an elastic part following Hooke's law and the plastic part following the usual plastic stress-strain relations. They establish four simultaneous equations for the unknown functions and their first derivatives.

These equations, however, are nonlinear, and the authors solve them numerically for certain boundary conditions. Even these numerical solutions are very difficult to obtain in the general case of a strain-hardening function, since the boundary conditions can only be satisfied by trial and error. The analysis becomes a good deal simpler if the true stress-true strain curve is a straight line in the elastic and a horizontal line in the plastic range, thus eliminating strain hardening. This case has been worked out for tubes with internal pressure and zero end load and ratios of the external to the internal radius of the tube between 1 and 4. The results of such computations in both regions are presented in eight plots of the radial, tangential, and axial stresses against the radius.

The authors refer to some previous work by Duguet, Macrae, and Nadai. The theory of Nadai agreed best with the paper's analysis, except for the axial stress distribution. The authors base their analysis in the plastic range on stress-strain relations in finite form. Such relations hold only if the ratios of the principal stresses do not change during the flow of an ideally plastic material (neglecting strain hardening). While the plastic zone advances through the wall of the cylinder, this condition, however, is not strictly satisfied. Thus the analysis of the authors must also be considered as an approximate solution of the flow in a partially yielded thick-walled cylinder.

A. Nadai, USA

978. N. F. Mott, "Slip at grain boundaries and grain growth in metals," *Proc. phys. Soc. Lond.*, Apr. 1948, vol. 60, pp. 391–394.

From his qualitative observations upon the behavior of metals at elevated temperatures Rosenhain proposed in 1910 that grain boundaries behave in a viscous manner with respect to a slow stress. The mechanical behavior of grain boundaries has ever since been a subject of controversy until last year when Kê, in a series of articles, demonstrated not only the validity of Rosenhain's concept, but actually measured the temperature dependence of the viscosity coefficient.

In the present paper the author proposes a detailed atomistic model which reproduces in a quantitative manner the observed temperature dependence. In contrast to older theories of viscosity, which assume as an elementary act the overcoming of some "potential barrier," the present theory proposes the elementary act to be essentially the melting of small regions at the grain interfaces.

Clarence Zener, USA

979. C. W. MacGregor, L. F. Coffin, Jr., and J. C. Fisher, "The plastic flow of thick-walled tubes with large strains," *J. appl. Phys.*, Mar. 1948, vol. 19, pp. 291–297.

The authors study the plastic flow of a thick-walled tube subjected simultaneously to internal pressure, external pressure, and an end load. True radial and tangential, rather than infinitesimal, strains are considered throughout. Starting from the strain compatibility equation and equilibrium equation for large deformations, together with a deformation stress-strain law of the Hencky-Nadai type and the assumption that the axial strain is constant over the cross section, the authors find the strain distribution in closed form, and the stresses in terms of simple quadratures. The case of a closed-end tube under internal and external pressure is studied in detail.

The finite strain theory as well as the infinitesimal strain theory, for a closed tube under internal pressure only, are compared with a pressure versus external tangential strain curve obtained experimentally. The infinitesimal strain theory predicts results far different from those of the finite strain theory and experiment for strains larger than 0.01.

G. H. Handelman, USA

980. K. Swainger, "The measurement and interpretation of post-yield strains," *Proc. Soc. exp. Stress. Anal.*, 1948, vol. 5, no. 2, pp. 1–8.

This paper discusses the use of Minalpha wire (Cu 85, Mn 12, Ni 3) as an electric-resistance strain gage to determine strains and stresses in the plastic range. The ultimate operating strain of this wire is as much as 14 per cent or even 25 per cent. Unfortunately the beginning of the load-deformation relationship is not straight. The author also discusses the interpretation of plastic strains obtained from a rosette in terms of stresses.

A. J. Durelli, USA

Design Factors, Meaning of Material Tests

(See also Rev. 965)

981. K. Wesołowski, "A study of steel beams from burned and destroyed structures (Badania belek stalowych ze spalonych i zburzonych budynków)," *Inżyn. Budown.*, July-Aug. 1947, vol. 4, pp. 337–340.

Tensile tests on specimens from steel beams recovered from burned and destroyed structures are discussed. The conclusion is reached that neither the temperature, nor the deformation during the fire and in the course of the cold straightening of the beams prior to re-use, have had any adverse effect on the structural qualities.

A. M. Freudenthal, USA

Material Test Techniques

(See also Revs. 939, 971)

982. T. R. Truax and M. L. Selbo, "Results of accelerated tests and long-term exposures on glue joints in laminated beams," *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 393–400.

A 21-day vacuum-pressure soaking-drying test procedure to accelerate the standard 180-day cyclic soaking and drying test is outlined with test results on laminated members indicative of actual durability in exterior service. The test cycle consists of immersion in water under 20 in. mercury vacuum for 2 hr followed by 2 hr of air pressure at 75 psi, both of which are repeated and followed by a 16-hr soak. The specimens are then dried for 6 days in a current of dry air. The 21-day test consists of 3 such cycles. The test is based upon the principle that severe shrinking and swelling of laminated members produce stresses which cause checking of wood, failure of glue joints, or both.

The test results are correlated with extensive exposure tests in a conclusive manner. Block shear tests conducted at various stages during the testing cycles were not consistent with joint performance under severe exposures.

E. A. Brittenham, Jr., USA

983. S. L. Anderson, "A method for obtaining stress-strain relations in nonisotropic flexible sheet material under two-dimensional stress," *J. sci. Instrum.*, Jan. 1947, vol. 24, pp. 25–27.

By means of internal-pressure tests with cylindrical envelopes of a rubberized sheet fabric, along with a uniform tension test on the same material, the author obtains data that enable him to

plot stress-strain curves for each of the two orthogonal directions, which give evidence of nonisotropic stress-strain properties. The "stress" or force per unit length is obtained from the relations for a long cylindrical envelope: circumferential stress = PR and axial stress = $^1/_2PR$, where P is the internal pressure and R the radius. The strains were measured mechanically.

Letting x and y be the two principal directions of the material, one point of the stress-strain curve for the x-direction (at a constant y-stress) was obtained by a cylinder test with the x-direction placed circumferentially; for another point the x-direction was axial. A third point was obtained from an ordinary tension test, the x-direction being lateral. Varying the values of y-stress yielded a family of these curves for the stress and strain in the x-direction. The ordinary tension test curve was used as a guide to the shapes of the entire family. The y-direction family was obtained similarly. Two of the four stress-strain quantities being known, the other two can then be obtained.

Julius Miklowitz, USA

984. **W. Lethersich, "Apparatus for the study of the rheological properties of dielectrics,"** *J. sci. Instrum.*, Mar. 1947, vol. 24, pp. 66–71.

Three optical strain gages designed for high sensitivity and accuracy (error = 0.10^{-5}) and high range (maximum strain = 0.12), which is required for work with plastics, are described. One gage is designed to measure axial strains, and a second one is capable of being mounted simultaneously to determine the accompanying lateral strain. The third gage is designed for use in a torsion test. All three gages make use of the optical lever principle. By using two mirrors the strain is a function of the deviation of the two reflected light rays, so that only one sighting telescope need be used for an entire series of gages. Application to creep tests is therefore very satisfactory.

The test results included furnish data on the modulus of elasticity, Poisson's ratio, and shear modulus of elasticity for several plastic materials.

Frank J. Mehringer, USA

985. **A. G. Ward and R. R. Marriott, "A constant stress apparatus for the study of the creep properties of plastics,"** *J. sci. Instrum.*, May 1948, vol. 25, pp. 147–151.

This paper describes the design and mechanical features of a constant stress tension-creep machine for plastics. The load is applied by dead weights hanging from a pulley of variable radius which corrects for the reduction of area of the specimen as it elongates. Calculations are given for one such pulley for a specimen test length of 10 cm, a Poisson's ratio of 0.5, and for specimen elongation of 50 per cent. Sample data obtained on this machine are presented.

Alexander Yorgiadis, USA

Mechanical Properties of Specific Materials

(See also Revs. 984, 985)

986. **F. Zalewski and W. Pogany, "Rubble as building material (Zużycie gruzu jako materiału budowlanego),"** *Inżyn. Budown.*, Jan. 1947, vol. 4, pp. 38–50.

Cement-bound rubble (cebetonite) is used for brick production in Poland. Tales are given of (1) its compression and bending strength; (2) the dependence of the bending strength on freezing and defreezing; and (3) its hygroscopic, drying, and thermal properties. A conclusion is reached that the quality of cebetonite equals that of ordinary bricks.

A. W. Wundheiler, USA

987. **W. Olszak, "Vibrating of concrete during the setting process (Wibrowanie betonu w czasie jego wiązania),"** *Inżyn. Budown.*, Mar.-Apr. 1947, vol. 4, pp. 177–182.

Compression and tension strength of concrete as a function of the time and procedure of vibration during setting have been experimentally obtained. A pronounced strength improvement, generally increasing with the length of the vibration process, has been noted.

A. W. Wundheiler, USA

Potential or Laminar Incompressible Flow

(See also Revs. 943, 1011, 1012, 1031, 1040, 1054, 1074, 1076, 1077)

988. **C. Ferrari, "On the theory of cascades of wing profiles (Sulla teoria delle schiere di profile alari),"** *R. C. Accad. Lincei*, Mar. 1947, vol. 2, pp. 289–299.

In a previous paper on the same subject, the author developed an approximate method for determining the aerodynamic characteristics of straight airfoils in cascade or lattice arrangement. The present paper extends this work to cambered profiles. Theories of Glauert, Betz, Pistolesi are reviewed, then conformal transformations for potential flow through cambered lattices are discussed, including conditions at the trailing edges. The influence of varying the angle of upstream approach is discussed, and a formula for optimum angle of attack is given. Consideration is given to the influence of finite thickness on the flow field. Finally, methods developed in the paper are applied to a cascade of NACA 8512 profiles as an example.

Frank L. Wattendorf, USA

989. **A. van der Neut and F. J. Plantema, "Impact loads on seaplanes during landing, with particular reference to the influence of the forward speed"** (in English), *Nat. LuchtLab. Amsterdam Rap.*, no. S.317, Oct. 1, 1947, pp. 1–11.

A three-dimensional solution, based upon strip theory, is developed for the water-load reaction in a seaplane landing. The assumptions are those commonly used in this problem. In defining the additional water mass no attempt is made to evaluate the effect of dead-rise angle on flat-plate coefficients, the effect of piled-up water, or the effect of aspect ratio. These effects are considered to be compensating, which is not seriously in error.

The nonlinear differential equation of motion appears to be adequately formulated. It is stated that numerical methods of solution were used, although no detailed explanation of the method is given. Graphs of the results, having general applicability, give the maximum acceleration plotted against a parameter which is the ratio of the tangent of the trim angle to the tangent of the initial flight path angle. If the sinking speed (initial vertical velocity) is assumed to be held constant, these graphs indicate the variation of maximum acceleration with horizontal velocity. No comparison with test data is given. The equations are developed for either flat or curved hull bottoms.

The differential equation for the three-dimensional case has been published, and solved numerically, for the V-bottom hull, by W. L. Mayo ["Analysis and modification of theory for impact of seaplanes," *Nat. adv. Comm. Aero. tech. Note*, no. 1008, 1945]. Many additional NACA technical notes and other reports have been published giving refined methods of solving the differential equation, solutions for curved bottoms, effects of wing elasticity, and experimental test results.

Stanley U. Benscoter, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 1030, 1047, 1062)

990. H. B. Squire, "Reconsideration of the theory of free turbulence," *Phil. Mag.*, Jan. 1948, vol. 39, pp. 1–20.

Free-turbulence problems consist of cases of turbulent flow in which walls play no part and for which the pressure variations are not important; wakes and jets are the most important of these. The author claims that the existing theories of free turbulence are unsatisfactory, because the distinction between dimensional analysis and physical hypotheses is not sharply drawn. Dimensional analysis is applied to the various problems of free turbulence in order to deduce as much as possible about the nature of turbulent flow without recourse to physical hypotheses. The various theories of turbulent flow are then considered in the light of this analysis.

It is concluded that the phenomena are too complicated to be explained by the simpler versions of the existing theories. Experiment shows that heat spreads faster than momentum, and this is qualitatively in favor of the vorticity transfer theory.

Wall turbulence is considered in an appendix and the logarithmic velocity distribution is deduced from dimensional analysis.

R. C. Binder, USA

991. A. E. von Doenhoff and Neal Tetervin, "Determination of general relations for the behavior of turbulent boundary layers," *Nat. adv. Comm. Aero. Rep.*, no. 772, 1943 (issued in 1948), pp. 1–25.

Analyzing the turbulent boundary layers of a series of NACA airfoils with different pressure distributions for Reynolds numbers from 0.90×10^6 to 4.18×10^6, the authors find that the velocity profiles in the boundary layer appear to be uniquely determined by a single parameter $H = \delta^*/\theta$ (δ^* = displacement thickness, θ = momentum thickness). This permits the development of a calculation method which is based on three equations containing δ^*, θ, the dynamic pressure q outside the boundary layer, and the skin friction per unit length τ_o.

The first equation is von Kármán's momentum equation

$$d\theta/dx + {}^1/_2(H + 2)\ (\theta/q)\ (dq/dx) = (\tau_o/2q)$$

For the second, an empirical relation is established between the dimensionless quantities $\theta(dH/dx)$ and $(\theta/q)\ (dq/dx)\ (2q/\tau_o)$, the latter being equivalent to Pohlhausen's parameter $(\delta^2/\nu)\ (dU/dx)$ for laminar boundary layers (δ = boundary-layer thickness). This relation is

$$\theta(dH/dx) = [\exp 4.680(H - 2.975)]\ [-\ (\theta/q)\ (dq/dx)\ (2q/\tau_o) - 2.035(H - 1.286)]$$

For the third equation the authors choose Squire and Young's empirical relation for the skin friction

$$2q/\tau_o = [5.890 \log_{10}(4.075\ R_\theta)]^2$$

After comparing this with Gruschwitz's method, the method is checked by application to some of the measured boundary layers, giving a reasonable agreement up to the region of boundary-layer separation occurring for values of H between 1.8 and 2.6. No systematic variation of the skin-friction coefficient with the shape parameter was found.

R. Timman, Holland

992. G. E. Nitzberg and S. Crandall, "Some fundamental similarities between boundary-layer flow at transonic and low speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1623, June 1948, pp. 1–30.

The applicability to the transonic shock intersection of the methods of analyzing the development of the boundary layer on a typical airfoil at low speed is considered. It is shown that there is a physical similarity between the low-speed laminar separation and transonic, lambda, laminar shock intersection. The low-speed relations are demonstrated to be qualitatively correct in indicating the reaction of the boundary layer when intersected by a shock. The Gruschwitz low-speed separation criterion is then applied to the turbulent layer shock intersection case, the apparent indication being that at stream Mach number above 1.5 separation will take place. This conclusion is apparently based on an assumed shock decelerating the flow from $M = 1.5$ to 1.0.

It is the reviewer's opinion that, aside from the question of the applicability of the Gruschwitz criterion, the assumption of a shock decelerating the flow from 1.5 to 1.0 is unrealistic, and that the authors' analysis indicates a critical Mach number of around 1.2 instead of 1.5.

C. B. Smith, USA

Compressible Flow, Gas Dynamics

(See also Revs. 992, 1004, 1008, 1010, 1013, 1016, 1017, 1018, 1019, 1032, 1034, 1035, 1038, 1041, 1044, 1045, 1065, 1072)

993. A. H. Stone, "On supersonic flow past a slightly yawing cone," *J. Math. Phys.*, Apr. 1948, vol. 27, pp. 67–81.

In this paper the flow parameters (velocity, pressure etc.) associated with the supersonic flow past a yawed cone are expanded in a perturbation series, using the unyawed (Taylor-Maccoll) flow as the zero-order flow, and the yaw angle of the cone as the perturbation parameter. The differential equations and boundary conditions governing the first-order terms are obtained and the solutions are then computed.

An interesting result which has been found by the author is that, to first-order terms, the head shock is a right cone also yawed to the incoming flow. The head force and ratio of shock yaw to cone yaw are computed for certain cases, and the latter are compared to available experimental data. The agreement is found to be excellent.

George Carrier, USA

994. M. J. Lighthill, "The hodograph transformation in transonic flow." *Proc. roy. Soc. Lond. Ser. A*, Nov. 18, 1947, vol. 191, pp. 323–369.

Exact solutions are obtained of the equations of motion for steady, isentropic and shock-free, two-dimensional perfect gas flows in symmetric channels, accelerated from rest to a uniform supersonic velocity. The stream function ψ is synthesized by a superposition (in the hodograph variables $w = \tau^{1/2}$, θ) of three component stream functions. The first is a finite sum $A_0\theta + \sum_1^m A_n\psi_n(w^2)\sin\ n\theta$. Here $\psi_n(w^2)$ is the hypergeometric function $w^nF(a_n b_n;\ n + 1;\ w^2)$, where $a_n + b_n = n - (\gamma - 1)^{-1}$, $a_n b_n = -\ (n/2)(n + 1)/(\gamma - 1)$, and γ is the specific heat ratio. The second has a simple pole in $(\partial\psi/\partial\theta)_{\psi=0}$ at the sonic velocity, so that the rate of change of velocity with distance along the channel axis ($\psi = 0$) is not zero at the sonic point. The third has a simple pole in $(\partial\psi/\partial\theta)_{\psi=0}$ at the velocity of the exist stream, so that the stream is uniform.

In part of the hodograph plane the function is found to be three valued, although single valued in the remainder. The two parts are separated by the two characteristic curves of the potential equation which emanate from the sonic point of the axis. All streamlines touch one or the other characteristic. An example of the design of a channel shape is given.

The properties of the hypergeometric function $\psi_n(w^2)$ are derived for complex n and asymptotic formulas for it are given.

A method is given which determines a uniform (and subsonic at infinity) flow around a body; this flow reduces to a chosen incompressible flow around a body of approximately the same shape when the Mach number becomes zero. The stream function is taken as $\psi = \sum_0^\infty c_n \psi_n f(n, w^2_\infty) e^{-in\theta}$, where w_∞ is the value of w in the free stream, and where the complex potential of the corresponding incompressible flow is given as $\Sigma c_n \zeta^n$ (ζ being the complex velocity). The function $f(n,w^2_\infty)$ is arbitrary when the flow is without circulation; with circulation only one of the solutions gives a closed body.

When the flow is entirely subsonic, the solution is valid in the entire hodograph plane. The velocity field of the incompressible flow need be known only near the body. When the flow contains supersonic regions, it is necessary that the incompressible flow data be given by the coefficients of a series $\Sigma d_n \zeta^n$ for the complex variable, where n may be negative and fractional. The subsonic solution can then be continued into the supersonic region.

W. G. Cornell, USA

995. **A. H. Shapiro and G. M. Edelman, "Tables for numerical solution of problems in compressible gas flow with energy effects,"** *J. appl. Mech.*, June 1948, vol. 15, pp. 169–175.

Tables are given for solving one-dimensional compressible gas flow problems. Energy effects which are represented by changes in stagnation temperature are taken into account. Tables are made for the specific heat ratios 1.0, 1.1, 1.2 1.3, 1.4, and 1.67. An example illustrates the use of tables.

Ahmed D. Kafadar, USA

996. **I. E. Garrick and Carl Kaplan, "On the flow of a compressible fluid by the hodograph method. Parts I and II,"** *Nat. adv. Comm. Aero. Rep.*, nos. 789 and 790, 1944 (issued in 1948), pp. 1–24 and 1–21.

No solution of the problem of compressible flow around a given body is attempted by the authors in these reports. In Part I, the authors are concerned mainly with the velocity and the pressure "correction formulas," i.e., the approximate formulas connecting the velocity and the pressure at a given point over the surface of the body at a certain free-stream Mach number to the corresponding values at zero free-stream Mach number. The basis of discussion is two functions $f(\tau)$ and $g(\tau)$, where τ is a parameter of velocity q defined by

$$W = \theta + i[\log q + f(\tau)]$$
$$i\tilde{W} = i[\theta + i(\log q + g(\tau))]$$

such that W and $\tilde{W}$ are two elementary solutions of the hodograph equations. θ is the inclination of the velocity vector to the x-axis. Since $[\log q + f(\tau)]$ and $[\log q + g(\tau)]$ occupy the same position as $\log q$ in the incompressible solution, it is proposed to equate them individually, and in certain combinations, to $\log q_i$, where q_i is the velocity of incompressible flow over a given body, and thus derive various correction formulas for velocity and pressure. This scheme is shown to contain various correction formulas previously proposed, such as von Kármán-Tsien and Temple-Yarwood [*Roy. Aircr. Estab. Rep.*, no. SME 3201, 1942]. The case involving the use of the arithmetic mean of $f(\tau)$ and $g(\tau)$ is worked out in detail and the correction for compressibility is larger than that of von Kármán-Tsien. The case involving the geometric mean of $f(\tau)$ and $g(\tau)$ is also discussed and is shown to coincide with the Prandtl-Glauert formula for thin bodies. The validity of any of these correction formulas is difficult to ascertain. The comparison with available accurate theoretical calculations and experiments seems however, to favor the von Kármán-Tsien formula.

Part II contains a detailed discussion of the elementary solutions for the potential functions of the form

$$\phi_k = P_k(q)^{\cos}_{\sin}(k\theta)$$

and the corresponding co-ordinates in the physical plane. It is shown that for such elementary solutions the limiting line will first occur at a local Mach number M between $M = 1$ and the value of M at the first zero of P_k in the supersonic region.

H. S. Tsien, USA

997. **Carl Kaplan, "On similarity rules for transonic flows,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1527, Jan. 1948, pp. 1–16.

Von Kármán's similarity rules for transonic flows, generalized by the inclusion of the ratio of specific heats as a parameter, is derived, using Tsien's method for obtaining similarity rules for hypersonic flows. Some investigation of the similarity parameter behavior as airfoil thickness approaches zero is included. The method suffers from the usual inadequacy that viscosity effects are not considered.

H. Julian Allen, USA

998. **Brown Univ., Grad. Div. Appl. Math., "Summaries of foreign and domestic reports on compressible flow, vol. IV,"** *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-1168D-ND, Nov. 1947, pp. 1–112.

This volume is a continuation of volumes I, II, and III of this series (see Revs. 502, 503, Applied Mechanics Reviews, Mar. 1948, p. 84). It contains 25 papers, the summaries of which are by various members of the staff of Brown University. Twenty sources are German, two American, two Italian, and one Swiss.

D. R. Mazkevich, USA

Aerodynamics of Flight; Wind Resistance

(See also Revs. 988, 989, 991, 1037, 1043, 1048, 1065, 1072)

999. **Stanley H. Scher, "Effect of 40 deg sweepback on the spin and recovery characteristics of a 1/25-scale model of a typical fighter-type airplane as determined by free-spinning-tunnel tests,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1256, Apr. 1947, pp. 1–26.

The results of this report indicate that under certain circumstances some improvement in spin recovery may be obtained by sweeping back the wings of a typical fighter airplane. A systematic evaluation of the effects of the various parameter changes associated with sweeping the wing was not made, thereby restricting the generality of the conclusions to the specific configurations tested.

J. M. Wild, USA

1000. **G. John, "A further development in calculating the 'take-off to 50-ft' distance of an airplane,"** *Aircr. Engng.*, Apr. 1948, vol. 20, pp. 98–101.

A rather thorough discussion is given of the calculation of a take-off distance which is divided into three arbitrary parts: a ground run, an accelerating period, and a climb-away period. The thrust is assumed to vary linearly with speed in order that the ground-run equation can be integrated. A derivation is given for the optimum lift coefficient to use during the ground run. The aircraft is assumed to fly horizontally during the acceleration period at an assumed altitude of 10 ft, no explanation being given of how the airplane suddenly attains that altitude with no change

in kinetic energy. The climb away is assumed to consist of a transition to a steady climb at the ultimate climbing speed. The equations are in such form that they can be adapted for other flight techniques. H. Reese Ivey, USA

1001. J. C. Sivells and R. H. Neely, "Method for calculating wing characteristics by lifting-line theory using nonlinear section lift data," *Nat. adv. Comm. Aero. tech. Note*, no. 1269, Apr. 1947, pp. 1–46.

This paper combines the methods of several authors into a single complete method of successive approximations for calculating wing characteristics using nonlinear section lift data. Simplified computing forms are presented, and a number of examples are worked out in detail. It is claimed by the authors that this method requires less computing time than most existing methods. Some numerical results are compared with experimental data, and with calculations made assuming linear section lift data. Somewhat better agreement between experiment and calculation is shown when the nonlinear data are used, particularly in the region of maximum-lift coefficient.

J. M. Wild, USA

1002. Macon C. Ellis, Jr., and Clinton E. Brown, "NACA investigation of a jet-propulsion system applicable to flight," *Nat. adv. Comm. Aero. Rep.*, no. 802, 1944 (publ. in 1948), pp. 1–17.

The results up to April 1943 of NACA experimental investigations and theoretical analyses of jet-propulsion systems suitable for flight are described. Specifically covered is an enclosed system, using a blower driven by an 825-hp radial Wasp engine and followed by chambers for fuel injection and combustion.

W. C. Johnson, Jr., USA

1003. F. A. Cleveland, "Afterburners for turbojet engines," *J. aero. Sci.*, May 1948, vol. 15, pp. 305–311.

A general discussion of means of thrust augmentation of turbojet engines is given. A method is presented for estimating the performance of any turbojet engine when equipped with an afterburner. The method is based on conventional thermodynamic and fluid dynamic formulas and presents little that is new.

Of interest is an application of the method to calculate the improved airplane performance through the use of an afterburner. The results of the example given for a four-engine 18,000-lb airplane show that the maximum speed near the transonic region at sea level is increased by 10 per cent. The sea-level rate of climb is increased by about 200 per cent and the normal take-off distance is halved when afterburners are employed.

The calculations also show that with unaugmented engines the take-off distance is increased by about 90 per cent when one engine fails during take-off, while for the case with turbojet engine using afterburners the take-off distance is only increased by about 27 per cent in such an emergency. C. A. Meyer, USA

1004. H. S. Ribner and F. S. Malvestuto, Jr., "Stability derivatives of triangular wings at supersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1572, May 1948, pp. 1–20.

An investigation of the theoretical stability derivatives of a triangular planform wing with vertex angle approaching zero at subsonic and supersonic speeds was given previously by one of the authors [*Nat. adv. Comm. Aero. tech. Note*, no. 1423, Sept. 1947]. The present report extends the analysis to triangular wings of arbitrary vertex angles at supersonic speeds but restricted to velocities for which the wing lies entirely within the Mach cone from its vertex. As long as this is true and the rotations are taken about the vertex, the shape of the pressure distribution for each motion is found to be the same as that for the low-aspect-ratio approximation.

It is shown that the results for the case of the triangular wing with vertex angle approaching zero for supersonic speeds can therefore be extended to the more general case, since the magnitudes differ by factors which are functions only of the ratio of the tangent of the semivertex angle of the triangle to the tangent of the Mach angle. Since the lift-curve slope and the damping in roll and pitch were determined elsewhere, this paper contributes primarily the side force and yawing moment derivatives and a tentative value of rolling moment due to yawing. All the known stability derivatives for this case are tabulated with respect to both body axes and stability axes. Irving C. Statler, USA

1005. Jack Fischel and Vito Tamburello, "Investigation of effect of span, spanwise location, and chordwise location of spoilers on lateral-control characteristics of a tapered wing," *Nat. adv. Comm. Aero. tech. Note*, no. 1294, May 1947, pp. 1–27.

This reports a wind-tunnel investigation of the effect of spoilers. The principal results were that: (1) The variation of rolling moment effectiveness with spoiler span showed a trend similar to that for ailerons; (2) the same was true for yawing moment effectiveness with spanwise location, but with opposite signs; (3) the effects are reduced with spoilers at the 80 per cent chord position, as compared with spoilers at the 50 per cent chord position. Wallace D. Hayes, USA

1006. Doris Cohen, "A theoretical investigation of the rolling oscillations of an airplane with ailerons free," *Nat. adv. Comm. Aero. Rep.*, no. 787, 1944 (publ. in 1947), pp. 1–12.

This paper is one of an NACA series on the dynamic stability of airplanes with one control free. It is a good basic reference, and supersedes the investigation of *Nat. adv. Comm. Aero. Rep.* no. 709.

The equations of motion are first written to include yawing and sideslipping, and it is demonstrated that the principal effects of freeing the ailerons can be determined without regard to these motions. If the ailerons tend to float against the wind and have a high degree of aerodynamic balance, rolling oscillations are likely to occur in addition to the normal lateral oscillations.

On the basis of equations including only the rolling motion and the aileron deflection, formulas are derived for the stability and damping of the rolling oscillations in terms of the hinge-moment derivatives and other characteristics of the ailerons and airplane. Charts are also presented showing the oscillatory regions and stability boundaries for a fictitious airplane of conventional proportions. The effects of friction in the control system are investigated and discussed. Conrad A. Lau, USA

1007. Thomas A. Toll and M. J. Queijo, "Approximate relations and charts for low-speed stability derivatives of swept wings," *Nat. adv. Comm. Aero. tech. Note*, no. 1581, May 1948, pp. 1–55.

Approximate relations are presented by the authors for the low-speed stability derivatives of swept wings. The analysis is made for wings without dihedral, but the authors consider the effects of sweep angle, of aspect ratio, and of center-of-gravity location. In most instances the effects of taper ratio are also considered. Several of the relations involve a correction factor (to account for the effects of sweep) and the derivative for an unswept wing which has the same aspect ratio and taper ratio as the swept wing.

The approximate relations have been used to construct charts for the stability derivatives of wings having a taper ratio of 1.0. The calculated values for the derivatives are compared with available experimental results. The comparison indicates that the calculated values are fairly reliable over a range of lift coefficient (starting from zero) which decreases as the sweep angle increases. Andrew Vazsonyi, USA

1008. **Kenneth Margolis, "Supersonic wave drag of sweptback tapered wings at zero lift,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1448, Oct. 1947, pp. 1–49.

On the basis of a recently developed theory for sweptback wings at supersonic velocities, equations are derived for the wave drag of sweptback tapered wings with thin symmetrical double-wedge sections, at zero lift.

Calculations of section wave-drag distributions and wing wave drag are presented for families of tapered planforms. Variations in drag with taper ratio, aspect ratio, sweepback, and Mach number are determined.

Calculations are also presented for families of tapered wings having equal root bending stress. H. P. Liepman, USA

1009. **Ralph W. Stone, Jr., and Sanger M. Burk, Jr., "Effect of horizontal tail position on the hinge moments of an unbalanced rudder in attitudes simulating spin conditions,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1337, June 1947, pp. 1–63.

This report contains the results of wind-tunnel tests in the NACA's Langley 15-ft free-spinning tunnel, made to determine the effect of horizontal tail location on rudder hinge moments in attitudes simulating spinning conditions. The tests were made on an aerodynamically unbalanced rudder for six positions of the horizontal tail and with the horizontal tail removed; tuft studies to determine the flow about the vertical tail were also made. The hinge moments were measured for angles of attack from 0 to 90 deg, and for angles of yaw from −30 to +30 deg; the rudder was deflected from 0 to 30 deg.

The results are presented in a form which makes them readily applicable to actual airplane design, and the pedal forces for several typical airplanes are calculated. It is pointed out that high pedal forces, such as are associated with the high forward position of the horizontal tail, do not necessarily mean poor recovery characteristics, since this position is generally indicative of good rudder effectiveness in spin recovery and large rudder deflections may therefore not be necessary. Conrad A. Lau, USA

1010. **H. Reese Ivey, "Notes on the theoretical characteristics of two-dimensional supersonic airfoils,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1179, Jan. 1947, pp. 1–23.

Two-dimensional calculations of aerodynamic characteristics at supersonic speeds are made by the shock expansion method on a series of double-wedge airfoils having different thickness ratios, locations of the maximum thickness, and cambers. In addition, a study is made of aileron size and effectiveness on a 5 per cent thick uncambered airfoil. The results are presented in the form of curves. A. W. Sherwood, USA

1011. **Edmund Pinney, "Aerodynamic forces on a slotted flat plate,"** *Quart. appl. Math.*, Apr. 1948, vol. 6, pp. 81–83.

Two-dimensional incompressible potential theory is applied to determine approximately the aerodynamic forces on a flat plate containing slots oriented parallel to the span. A velocity-distribution function is constructed which satisfies the boundary conditions of flow at all points on the plate, including the Kutta condition at the trailing edge of each segment formed by the spanwise slots.

It is concluded that a flat-plate airfoil, slotted parallel to the span symmetrically about its mid-chord, has the same steady-state aerodynamic lift and moment as an unslotted flat plate of chord equal to the total chord of the slotted plate minus the total width of the slots.

It appears to the reviewer that the assumption of potential flow for all segments of the slotted airfoil is not reasonable when the width of the slots is small compared to the width of the segments. Edward N. Bowen, USA

1012. **Charles E. Watkins, "The streamline pattern in the vicinity of an oblique airfoil,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1231, Mar. 1947, pp. 1–26.

This report outlines a method for determining the streamline flow pattern of a nonviscous incompressible fluid about an oblique airfoil from the corresponding flow pattern about the airfoil in the normal position. It is pointed out that, in its practical application, the effect of the boundary layer can be expected to be more significant than in the normal case. It is also noted that approximate compressibility effects can be readily obtained by applying the Prandtl-Glauert correction factor to the flow pattern normal to the leading edge.

The report will be useful in applied aerodynamics in determining the optimum shape, for a given lift coefficient, of a nacelle or fuselage intersection on a swept wing, to minimize the adverse interference effects of these bodies on the beneficial effects of sweep at high Mach numbers. Conrad A. Lau, USA

1013. **Robert T. Jones, "Subsonic flow over thin oblique airfoils at zero lift,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1340, June 1947, pp. 1–20.

In this report a distribution of line sources is set up to correspond to the three-dimensional flow over thin airfoils at zero lift. The pressure field produced by a line source is shown to be equivalent to the sum of two conical fields radiating from the ends of the line source, thereby considerably simplifying the treatment.

The analysis is applied to straight and sweptback wings of wedge and biconvex sections to show typical pressure distributions at the tip, root, and intermediate sections. The pressure distribution at the root of a sweptback wing is shown to deviate considerably from the two-dimensional form. This result is of importance in the prediction of compressibility effects at high subsonic speeds. A. W. Sherwood, USA

1014. **Flight Research Maneuvers Section, Langley Memorial Aero. Lab., "Flight studies of the horizontal tail loads experienced by a fighter airplane in abrupt maneuvers,"** *Nat. adv. Comm. Aero. Rep.*, no. 792, 1944 (publ. in 1947), pp. 1–16.

The airplane tested had a wing span of 40 ft and gross weight of 12,000 lb. In unstalled flight, pressures were measured at a few points on the surface and the flight tail load was computed by using the pressure distribution found in full-scale wind-tunnel tests of the same airplane. In stalled flight, the loads were determined from tip deflections obtained photographically, using the influence line for static loads and the wind-tunnel pressure distribution.

Abrupt pull-ups to maximum lift were made from minimum speed to 214 mph, with corresponding normal accelerations of 1.0 g to 4.5 g. Some 180-deg turns were also made. Continuous records of air speed, elevator angle, stick force, and normal acceleration were obtained.

For unstalled pull-ups the critical load was upward and was approximately proportional to maximum normal acceleration. In power-on unstalled maneuvers the spanwise loading was unsymmetrical, the dissymmetry being independent of normal acceleration. In power-off maneuvers the dissymmetry was greatly reduced. In stalled pull-up maneuvers a large abrupt increase in tail load was noted just after wing stall, with repeated buffeting by the turbulent flow from the stalled wing, which sometimes approached the natural frequency of the stabilizer in bending or the fuselage in torsion and caused large deflections.

John T. Kirkland, USA

1015. R. MacLachlan and W. Letko, "Correlation of two experimental methods of determining the rolling characteristics of unswept wings," *Nat. adv. Comm. Aero. tech. Note*, no. 1309, May 1947, pp. 1–23.

Rolling characteristics (rate of change of rolling moment coefficient with wing-tip helix angle and rate of change of aileron hinge-moment coefficient with wing-tip helix angle) of several wings were determined: (1) by rotating the wing in straight flow; and (2) by holding the model stationary in rotating flow. Angles of attack and of yaw were varied.

Values of the two lateral stability derivatives obtained by the two methods agreed with each other and with calculations. When the wing was yawed the derivatives were approximately proportional to the square of the cosine of the angle of yaw, as might be expected.

John T. Kirkland, USA

1016. John C. Evvard, "The effects of yawing thin pointed wings at supersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1429, Sept. 1947, pp. 1–26.

This report applies the excellent method of the same author [*Nat. adv. Comm. Aero. tech. Note*, no. 1382, July 1947] on linearized supersonic wing theory to the problem of a wing with no supersonic leading edge. Unfortunately an approximation is necessary which prevents accurate quantitative results, although permitting some qualitative conclusions. As a result the great essential versatility of the method is lost in this particular application.

For the yawed delta wing, for which the correct quantitative solution must be obtained by other methods, the author concludes from his analysis that "the center of pressure of the yawed delta wing lies along a line parallel to the free-stream direction passing through the center of the delta base and at a distance two thirds of the maximum chord aft of the vertex."

Wallace D. Hayes, USA

1017. Arthur L. Jones and Alberta Alksne, "The damping due to roll of triangular, trapezoidal, and related planforms in supersonic flow," *Nat. adv. Comm. Aero. tech. Note*, no. 1548, Mar. 1948, pp. 1–23.

In this note the authors use the surface integral methods based on linearized potential theory to obtain the damping in roll for several planforms. The planforms investigated are triangular, trapezoidal, and rectangular. Two sweptback forms are also considered.

The principal results obtained indicate that as the aspect ratio is increased, the limiting value of the damping in roll for trapezoidal and rectangular planforms equals the value for two-dimensional flow. This is twice the limiting value for triangular planforms.

In the case of sweptback planforms whose leading edges are coincident to the Mach cone, the damping in roll exceeds the value for two-dimensional flow. An investigation of the effect of reversal of the planform position relative to the stream direction revealed the fact that this reversal had no effect on the value of the stability derivative for damping in roll.

Louis A. Pipes, USA

1018. W. F. Lindsey, B. N. Daley, and M. D. Humphreys, "The flow and force characteristics of supersonic airfoils at high subsonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1211, Mar. 1947, pp. 1–56.

The paper reports an investigation conducted in the Langley rectangular high-speed tunnel on five supersonic airfoils and (for comparison) two subsonic airfoils, at angles of attack from 0 to 4 deg, for Mach numbers between 0.30 and 0.90.

Curves are presented showing the effect of angle of attack and Mach number on the pressure distributions, section normal force, pitching moment, and drag coefficient. Schlieren photographs show the variation of the flow phenomena with angle of attack and Mach number.

The results indicate that the drag coefficients are generally higher at subsonic Mach numbers for the supersonic airfoils than for the subsonic airfoils, but that the normal force and pitching-moment characteristics of the supersonic airfoils having their maximum thickness located at the 0.7-chord station would diminish the generally encountered problem of longitudinal control at high Mach numbers.

The investigation also revealed the occurrence of an unusual flow phenomenon at the leading edges of the supersonic airfoils at the high Mach numbers, which may have an appreciable effect on the maximum lift coefficient of supersonic airfoils at high subsonic Mach numbers.

Nicholas Di Pinto, USA

1019. Leonard Sternfield, "Some considerations of the lateral stability of high-speed aircraft," *Nat. adv. Comm. Aero. tech. Note*, no. 1282, May 1947, pp. 1–54.

In this paper the author investigates the effect of directional stability and effective dihedral on the dynamic longitudinal stability of a supersonic-type airplane. The stable and unstable ranges of these variables are established and the general trends of the other variables are indicated.

The analysis is conventional, the problem being treated as a vibration in three degrees of freedom with aerodynamic and mass coupling.

One of the most interesting points brought out by this particular analysis is the importance of the inclination of the longitudinal principal axis to the flight path.

C. B. Smith, USA

1020. William H. Phillips, "Application of spring tabs to elevator controls," *Nat. adv. Comm. Aero. Rep.*, no. 797, 1944 (issued in 1948), pp. 1–14.

The equation for the control stick force of an airplane having a spring-tab-equipped elevator is derived for the case of steady pull-ups; it is obtained from the elevator hinge moment (with due allowance for the effect of tab deflection), the tab's own hinge moment, and the force due to deflection of the spring.

It is shown that at low speeds the control force characteristics with the spring tab approach those of the corresponding elevator without such a tab, while at high speeds they approach those of an elevator actuated by a pure servo tab. A variation of stick force thus takes place as the airplane speed changes, which has been found undesirable. This may be reduced by reducing the stiffness of the spring, or by using a combination geared (balancing) tab and spring tab. The first method may give too light control forces for small airplanes, and too "soft" a control for ground han-

dling. The latter method reduces undesirable force variation since the geared tab reduces the forces and a less powerful tab is required.

The results of calculations for four typical airplanes weighing from 16,000 to 500,000 lb show that the spring tab offers a possible solution to the problem of acceptable elevator control forces for such airplanes. W. O. Breuhaus, USA

1021. **L. A. Simonov, "Calculation of an airfoil in a flow and mapping of an airfoil according to a distribution of velocities over its surface"** (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1947, vol. 11, pp. 69–84.

The paper is based on the classical method of conformal mapping of a circle onto an airfoil by means of a function of the type

$$Z(\zeta) = a + b\zeta + \sum_n c_n/\zeta^n$$

The author considers first the problem of finding a profile corresponding to an assigned function $y(\theta)$ (or $dy/d\theta$), where in the plane of the profile $z = x + iy$ and in the plane of the circle $\zeta = r \exp(i\theta)$. An approximate method of solution of these problems is given, consisting in the determination of changes which must be imparted to a profile of known characteristics in order that y and $dy/d\theta$ take the assigned values.

A similar method is used for the determination of a profile corresponding to an assigned distribution of pressure. The latter must satisfy certain conditions in order that the profile be a closed curve not intersecting itself. In a similar manner the author determines changes of flows around profiles subject to small changes, and the effect of linear superposition of conformal mappings. Several numerical tables facilitating the application of the method are considered. I. Opatowski, USA

1022. **R. B. Bland and T. D. Reisert, "An application of statistical data in the development of gust-load criterions,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1268, Apr. 1947, pp. 1–26.

This paper proposes a method for analyzing known statistical data on the frequency of occurrence of atmospheric gusts in terms of the number of critical gusts that will be encountered by an airplane during its operational life. This provides a more rational basis for estimating gust-load factors for structural design than is used in current design practice.

To apply this method it is necessary to know or to assume for any given airplane its speed-time distribution, the permissible number of critical gusts to be encountered during its operational life, and the operational life of the airplane in miles to be flown. The authors have applied this method to several representative airplanes by assuming a range of reasonable or limiting values of these parameters, and they compare the results with present design requirements. They conclude that under certain circumstances the present requirements are inadequate or unreasonable. J. M. Wild, USA

1023. **Harry Greenberg and Leonard Sternfield, "A theoretical investigation of longitudinal stability of airplanes with free controls including effect of friction in control system,"** *Nat. adv. Comm. Aero. Rep.*, no. 791, 1944 (issued in 1948), pp. 1–21.

This reports an investigation of factors related to the longitudinal stability of airplanes with controls free. Both static and dynamic stability analyses are employed in the study. The classical equations of motion developed by Bryan and Bairstow, extended to cover the effects of control deflections and their coupling with the airplane motions, are used as the basis of the analyses.

In the static stability analysis the stick-force gradients with speed and normal acceleration are expressed as functions of the airplane and control parameters. These parameters include the restoring tendency, floating tendency, mass unbalance, and moment of inertia of the elevator control system plus the density and center-of-gravity location of the airplane. Graphical results based on calculations for a typical airplane show the variation with the parameters investigated of the stick-force gradients, which are, incidentally, standard stability criteria for flying qualities requirements.

The dynamic stability analysis includes the effect of adding viscous friction and solid friction to the control system, in addition to the effects of the parameters investigated in the static stability analysis. Stability boundaries are established for the short-period oscillations and aperiodic modes. The stability of the short-period oscillations is the primary consideration here, the long-period oscillation (phugoid motion) being considered unimportant to this investigation.

Suggestions for improving the static stability characteristics are given. The dynamic stability without friction in the control system is considered in the light of remedial action for improving the stability of the short-period oscillations. A discussion is included of the beneficial effects of viscous friction provided the center of gravity is not behind a certain critical position, and of solid friction under conditions where the viscous friction causes increasing oscillations. Arthur L. Jones, USA

1024. **Henry A. Pearson, "Derivation of charts for determining the horizontal tail load variation with any elevator motion,"** *Nat. adv. Comm. Aero. Rep.*, no. 759, 1943 (issued in 1948), pp. 1–15.

In this paper the equations relating the wing and tail loads are derived for a unit elevator displacement. These equations are then converted into a nondimensional form, and charts are given by which the wing- and tail-load-increment variation may be determined under dynamic conditions for any type of elevator motion and for various degrees of airplane stability.

In order to illustrate the use of the charts, several examples are included in which the wing and tail loads are evaluated for a number of types of elevator motion. Methods are given for determining the necessary derivatives from the results of wind-tunnel tests when such tests are available.

Conrad A. Lau, USA

1025. **J. Fischel and M. F. Ivey, "Collection of test data for lateral control with full-span flaps,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1404, Apr. 1948, pp. 1–180.

This report presents in a consistent system of nomenclature a collection of the essential data from previously published reports on wind-tunnel and flight tests with various combinations of full-span flaps and different types of lateral-control devices. The rolling effectiveness of the various configurations of flap and control device and the hinge moments for the control device are discussed. Other characteristics, such as yawing tendencies, are also discussed whenever pertinent. John T. Kirkland, USA

1026. **D. R. Mulholland and P. J. Perkins, "Investigation of effectiveness of air heating a hollow steel propeller for protection against icing; I. Unpartitioned blades; II. 50 per cent partitioned blades; III. 25 per cent partitioned blades,"** *Nat. adv. Comm. Aero. tech. Note*, nos. 1586, 1587, 1588, May 1948, pp. 1–51; 1–25; 1–23.

Test results are reported on the use of hot air for the de-icing of propeller blades. Hot air was introduced near the hub of a

hollow steel blade, passed radially out through the blade, and discharged near its tip. Tests were performed in an icing tunnel.

In the first report test results obtained without internal radial partitions are presented and show that only a small portion of the heat passing through the blade is effective in raising the temperature of the leading edge. In the tests described in the second report a radial internal partition permitted the passage of hot air through the leading edge half of the blade only. This provided good protection against icing at considerably lower heat energy inputs. For the third report only the leading edge one quarter of the interior of the propeller blade was heated, and the most effective use was made of the heat added; however, the passage had become so small that insufficient air could be passed through to prevent icing under the most severe conditions.

Howard W. Emmons, USA

1027. Robert T. Jones and Harry Greenberg, "Effect of hinge-moment parameters on elevator stick forces in rapid maneuvers," *Nat. adv. Comm. Aero. Rep.*, no. 798, 1944 (issued in 1948), pp. 1–8.

It has not been unusual to consider stick forces in steady rectilinear and steady accelerated flight as the major criteria for satisfactory elevator stick force characteristics. This report is concerned with additional requirements for satisfactory elevator stick forces. It is shown from a theoretical analysis that widely different transient stick forces for rapid maneuvers will be obtained for the same steady-state stick force per "g" value, since this value can be achieved by widely different mass and aerodynamic parameters.

The analysis is made for constant speed with an assumed variation of elevator deflection with time. High negative values of the restoring tendency C_{h_δ} and the floating tendency $C_{h_{\alpha_t}}$ may lead to a high initial value of stick force followed by a reversal, in abrupt maneuvers. This difficulty can be avoided by reducing C_{h_δ} and making $C_{h_{\alpha_t}}$ slightly positive, but there is a lower limit beyond which C_{h_δ} cannot be reduced to prevent the initial stick force from being too light. The effect of a bob weight is considered, and it is shown to be similar to a change in $C_{h_{\alpha_t}}$ except for a phase difference. It is interesting to note that flight experience has verified, in general, the conclusions that may be drawn from this report.

Conrad A. Lau, USA

1028. A. Gessow, "Standard symbols for helicopters," *Nat. adv. Comm. Aero. tech. Note*, no. 1604, June 1948, pp. 1–13.

A list of standard symbols for use in studies of helicopter aerodynamics has been prepared under the sponsorship of the SAE and NACA. The reference axis for air flow relative to the rotor is taken to be the "axis of no feathering," so that these parameters are applicable to older types of helicopters in which no provision for feathering was made. John E. Goldberg, USA

1029. Pierre Périlhou, "New method of blind landing (Nouveau procédé d'atterrissage sans visibilité)," *Rech. aéro. Paris*, Mar. 1948, no. 2, pp. 7–20.

A new method of making blind landings is described which requires much less equipment on the ground and in the airplane than other "unattended" systems. The description is preceded by an appraisal of other systems.

For airplane heading this new system uses a constantly rotating narrow radio beam of very short wave length co-ordinated with a nondirectional flash signal. For distance, two radio receivers are used, one at each wing tip, and the time for the rotating beam to traverse the distance between them indicates the distance to the beacon. For altitude a standard electronic altimeter is recommended. The amount of cross wind can be indicated by installing another receiver in the tail, to indicate by the displacement of the tail to the left or right how much the airplane is turned to face the relative wind. The information is shown on an oscilloscope and the pilot is able to adjust his path for varying conditions.

W. C. Johnson, Jr., USA

1030. S. F. Racisz and J. F. Cahill, "Wind-tunnel investigation of effects of forward movements of transition on section characteristics of a low-drag airfoil with a 0.24-chord sealed plain aileron," *Nat. adv. Comm. Aero. tech. Note*, no. 1582, May 1948, pp. 1–26.

A 12 per cent thick low-drag airfoil with a 0.24-chord sealed plain aileron was tested in a two-dimensional flow low-turbulence wind tunnel at a Reynolds number of 14×10^6. The Mach number during the tests was never greater than 0.17. The section lift, aileron section hinge-moment, and seal-pressure characteristics were determined with aerodynamically smooth surfaces, with transition fixed at 0.30 chord, and with transition fixed at the airfoil leading edge, for aileron deflections between −20 and +20 deg. Neal Tetervin, USA

1031. M. M. Callan, "The vertical component of induced velocity in the plane of symmetry of an aeroplane," *Nat. Res. Counc. Can. aero. Note*, no. 1, 1948, pp. 1–8.

Based upon Glauert's expression for the downwash ratio, curves are drawn for the normal components of the induced velocity in the plane of symmetry in the neighborhood of a tailless airfoil having an aspect ratio of 6 and flying at $C_L = 1.0, 2.0$. Charts are also presented which simplify the calculation of the flow pattern for any values of aspect ratio and lift coefficient.

It is suggested that, with these curves, a suitable relative location may be selected for a trailing bomb for measuring flight path and true air speed of such aircraft as tailless gliders.

John E. Goldberg, USA

1032. H. Reese Ivey and Edward N. Bowen, Jr., "Theoretical supersonic lift and drag characteristics of symmetrical wedge-shape airfoil sections as affected by sweepback outside the Mach cone," *Nat. adv. Comm. Aero. tech. Note*, no. 1226, Mar. 1947, pp. 1–60.

Lift and wave drag are computed for the two-dimensional flow regions of untapered wings, of symmetrical double wedge cross section, swept back outside of the Mach cone. The effect of friction is approximated by use of a friction-drag coefficient of 0.006. Three families of wings are discussed, some having a ratio of maximum thickness to chord of 0.05, while others have ratios of maximum thickness to effective chord (projection of chord on normal to leading edge) of 0.05 and 0.10. The data cover the range of Mach numbers from 1.5 to 8.0 and sweepback angles from 0 to 60 deg. W. G. Cornell, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 1020)

1033. A. R. Collar, "Semirigid representation of elastic cantilevers for torsion problems in aerodynamics," *Phil. Mag.*, Feb. 1948, vol. 39, pp. 97–104.

Evidently intended primarily for readers unfamiliar with Rayleigh's principle, the paper develops it in connection with the problem of wing torsional divergence, thereby illustrating the

usefulness and demonstrating the legitimacy of the semirigid representation. S. B. Batdorf, USA

1034. **E. A. Krasilshchikova, "Disturbed motion of air caused by vibration of a wing moving at supersonic speed"** (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Jan.-Feb. 1947, vol. 11, pp. 147–164.

The author considers the problem of a vibrating thin wing, normal to the z-axis and moving at supersonic speed in the x-direction. The velocity potential $\varphi = \varphi^{(o)}(x,y,z) + \varphi^{(1)}(x,y,z,t)$, where $\varphi^{(1)}$ is the transient potential, is assumed to satisfy the linearized equation $(M^2_o - 1)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} - \varphi_{tt} + 2M_o\varphi_{xt} = 0$. The flow is assumed to be symmetric with respect to the x-y plane. Behind the wing an infinite half-strip of discontinuity Σ in the x-y plane is assumed, along which the values of φ_z, $\varphi_x^{(0)}$ and $(\varphi_t^{(1)} - u\varphi_x^{(1)})$ are equal for $z \to +0$ and $z \to -0$. On the intersection S of the wing with the x-y plane we have $\varphi_z^{(0)} = A_o(x,y)$, $\varphi_z^{(1)} = Re A_1(x,y)e^{i\omega t}$, where $z = A_o(x,y)$ is the equation of the boundary of the wing, and $A_1(x,y)$ and ω are given quantities. Along the Mach surface of the leading edge and finally in the domain σ bounded by the Mach surface and the discontinuity strip, we have $\varphi(x,y,z,t) = 0$.

The author assumes the solution $\varphi^{(1)}$ of the form $\varphi^{(1)} = Re F(x,y,z)e^{\beta x + i\omega t}$, where F satisfies a linear equation and corresponding boundary conditions. She considers at first a wing of infinite span, in which case the equation for F assumes the form $F_{zz} - F_{xx} - \lambda^2 F = 0$, which equation is solved by Riemann's method. She obtains for F an expression which in the special case $\lambda = 0$ coincides with the formula obtained previously by Ackert.

Using the results obtained for the plane problem she obtains by means of Fourier series the expression

$$F(x,y) = \frac{1}{\pi}\iint\limits_{\sigma(x,y)} \frac{A(\xi,\eta)\cos\left[\lambda\sqrt{(x-\xi)^2 - (y-\eta)^2}\right]}{(x-\xi)^2 - (y-\eta)^2}\, d\eta d\xi$$

for a wing of finite span. If the leading edge of the wing is given by $x = \psi(y)$, the trailing edge $x = \chi(y)$, the assumption is made that on the contour the values $\partial\psi/\partial y$ and $\partial\chi/\partial y$ do not exceed the cotangent of the Mach angle. The solution of a finite wing with arbitrary leading edge is reduced to an integral equation

$$\iint\limits_{\sigma(\xi,\eta)} \frac{\theta(\xi',\eta')\cos\left[\lambda\sqrt{(\xi-\xi')^2 - (\eta-\eta')^2}\right]}{\sqrt{(\xi-\xi')^2 - (\eta-\eta')^2}}\, d\eta' d\xi' = D(\xi,\eta)$$

where D is a known function.

Finally the author gives the solution of the second basic problem in the theory of vibrating wings: Assuming the distribution of pressure over a wing, to determine normal components of the velocity and to determine the form of oscillation.

Stefan Bergman, USA

1035. **S. N. Karp, S. S. Shu, and H. Weil, "Aerodynamics of the oscillating airfoil in compressible flow,"** *Hdqtrs. Air. Mat. Comm. Dayton tech. Rep.*, no. F-TR-1167-ND, Oct. 1947, pp. 1–56.

An integrated survey of the linearized two-dimensional theory of oscillating thin airfoils in compressible flow is presented in this monograph. The basic differential equation for unsteady flow with small disturbances is derived. Boundary conditions and methods of solution for the subsonic and the supersonic problems are discussed in separate chapters. The treatment of the subsonic case begins with a derivation of Possio's integral equation for the pressure jump at the oscillating airfoil. Numerical methods of solution due to Schade and to Dietze are discussed in some detail.

In the supersonic case with harmonic oscillations the basic equation is transformed into a hyperbolic differential equation with constant coefficients. Several methods of solution are discussed, and an integral representation is derived for the lift distribution on the oscillating airfoil. Formulas are given for lift and moment due to rotational and translational oscillations in terms of functions which have been tabulated by von Borbély, Temple and Jahn, and Garrick and Rubinow. An extensive bibliography is included. M. J. Turner, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 988, 1003, 1026, 1057, 1059, 1060)

1036. **B. Eckert, F. Pflüger, and F. Weinig, "The influence of the diameter ratio on the characteristics diagram of the axial compressor,"** *Nat. adv. Comm. Aero. tech. Memo.*, no. 1125, Apr. 1948, pp. 1–57 (transl. from publ. of *ForschInst. Kraftfahrwes. Fahrzeugmot. T. H. Stuttgart*).

The authors investigate the influence of the diameter ratio of axial blowers on efficiency and pressure coefficient. The diameter ratio becomes of great importance for highly loaded axial-flow compressors. This analysis is of interest not only for single-stage units but for highly loaded multistage units presently used in aircraft power plants. The test data reported are for a single-stage blower only, showing the attainable maximum-pressure coefficient and optimum efficiency as a function of the diameter ratio and as a function of specific speed and throttling coefficient. The results show a decrease in maximum-pressure coefficient and optimum efficiency with increase in diameter ratio, which is ascribed to the drag effect of the hub surface and housing.

In addition a theoretical analysis is presented dividing the overall efficiency into partial efficiencies and showing the change in each partial efficiency caused by the diameter ratio. This report contains valuable data for the case investigated, i.e., a single solidity and only one Reynolds number. H. E. Sheets, USA

1037. **Frank W. Davis, "Problems of gas-turbine-propeller combinations,"** *Aero. Engng. Rev.*, Apr. 1948, vol. 7, pp. 30–36.

The paper gives a detailed account of operation on the ground and in flight with the TG-100 turboprop engine. Two types of propeller and control were used in the flight tests and serious control problems were encountered with each. The author emphasizes that the flight tests were achieved by "stop-gap" solutions for the propeller-engine control problems. As of that date, December 1945, the conclusion is inescapable that satisfactory propeller-engine controls for the turbopropeller type did not exist.

The author suggests that the propeller governor might better respond to torque rather than to rpm and that a temperature-sensitive topping governor be developed to prevent excessive turbine temperature under any conditions. Others have suggested that in view of the essentially constant-speed-type operation characteristic of turbines the propeller governing should be tied to the temperature governing. John Stack, USA

1038. **A. Ginsburg, W. K. Ritter, and J. Palasics, "Effects on performance of changing the division of work between increase of angular velocity and increase of radius of rotation in an impeller,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1216, Feb. 1947, pp. 1–27.

The ideal work of compression in a centrifugal-type impeller is

obtained: (*a*) by angular acceleration of the air in the inducer section, and (*b*) by increased air velocities resulting from an increased radius of rotation in the radial-bladed section. Conventional impellers are designed to have most of the work of compression done by (*a*), because this is believed to be most efficient. To clarify this question, tests were made on four impellers all having the same inducer section but various amounts of area of the radial blades. The results were as follows:

1 The impeller adiabatic efficiency, pressure coefficient, and volume-flow capacity were improved when the amount of work resulting from angular acceleration was increased with respect to the work resulting from an increased radius of rotation.

2 The volume flow was independent of impeller diameter and tip speed but increased linearly with impeller angular velocity.

3 A volume-flow restriction occurred in the inducer section when the pressure dropped to a critical value, which occurred at lower relative Mach numbers as the blade inlet angles of attack decreased.

4 Any detrimental effects on impeller performance resulting from transonic velocities were small compared to the desirable performance effects obtained by increasing angular velocity and thus increasing the work of compression by angular acceleration.

Karl E. Schoenherr, USA

1039. A. I. Ponomareff, "Axial-flow compressors for gas turbines," *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 295–306.

The apparent purpose of this paper is to introduce the axial-flow compressor to engineers unacquainted with gas turbines. The discussion covers briefly practically all phases of axial-flow-compressor design and operation including: gas-turbine requirements, characteristic compressor curves, stability limits, possible blade arrangements, effects of velocity ratio, pitch chord ratio, aspect ratio, Reynolds number, Mach number, and test methods.

John V. Becker, USA

1040. Andrew Vázsonyi, "On the aerodynamic design of axial-flow compressors and turbines," *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 53–64.

This paper offers a theoretical method for the determination of the flow of a perfect incompressible fluid through a cascade of compressor blades, but the method is applicable also to turbines and axial (helical) pumps.

The lift coefficient can be determined as a function of the angle of attack within ten hours. The complete study of all the cascades using the same profile would take sixty hours. The procedure is graphical and does not require a knowledge of mathematics. The author claims excellent agreement with experience. An extension of the method permits us to determine pressure and velocity distributions.

A. Tenot, France

1041. Frank L. Wattendorf, "High-speed flow through cambered rotating grids," *J. aero. Sci.*, Apr. 1948, vol. 15, pp. 243–247.

The paper deals with adiabatic flow of ideal compressible fluids through a cascade of rotating blades. The problem has been treated for an incompressible flow in helical pumps, Kaplan wheels, and axial fans. The results of these theoretical computations have been well confirmed by experiment. The author presents similar computations for compressible flow in axial compressors in the following four cases: (1) Subsonic flow throughout; (2) supersonic flow in the downstream stator only; (3) supersonic flow in rotor and in stator, but subsonic axial flow; (4) supersonic flow throughout.

Stage temperature or stage pressure ratios replace the ordinary incompressible pressure coefficient and have the same significance; the latter referring to the "stagnation velocity at infinity." The temperature head can be determined from the velocity diagrams using Euler's equations.

The results of this study show very large values of the compression gradient possible in axial compressors. Axial compressors require a greater length than centrifugal ones, because to realize compression gradients of the order of 3-4 a large number of stages are necessary (provided that the gradient is limited to 1.25 per stage, that is, if the flow is subsonic). The author shows that by using highly cambered blades the compression gradient per stage can be substantially increased if efficiency is ignored (the flow is assumed to be adiabatic).

A. Tenot, France

1042. R. J. Anderson, W. K. Ritter, and S. R. Parsons, "Apparent effect of inlet temperature on adiabatic efficiency of centrifugal compressors," *Nat. adv. Comm. Aero. tech. Note*, no. 1537, Feb. 1948, pp. 1–18.

Many investigators have found a variation in adiabatic efficiency of centrifugal compressors at constant equivalent tip speed and equivalent volume flow, due to variation in inlet air temperature—an effect for which there is no theoretical basis if adiabatic compression is assumed. This paper reports an investigation to determine whether this apparent effect could be reduced or eliminated. Three compressor arrangements were tested at various equivalent tip speeds, inlet temperatures, and equivalent volume flows.

The effect of varying Reynolds number over a greater range than would result from an inlet-temperature spread of 150 F resulted in relatively small discrepancies in compressor adiabatic efficiency.

Calculations of heat-transfer effects were then made. By using high recovery thermocouples at the diffuser discharge to measure compressor air discharge temperature, very consistent results were obtained. It is concluded that if efficiency is based on compressor discharge temperature measured where external heat transfer has had negligible opportunity to occur, the apparent variation of adiabatic efficiency is eliminated.

E. G. Allen, USA

1043. Walter Castles, Jr., and Arnold L. Ducoffe, "Static thrust analysis for helicopter rotors and airplane propellers," *J. aero. Sci.*, May 1948, vol. 15, pp. 293–299.

General formulas are derived for the thrust and torque coefficients of a helicopter rotor in hovering, or of a propeller. These formulas are in the form of definite integrals which can be evaluated numerically or graphically for any given data. They are valid for tapered and twisted blades, although explicit expressions are worked out for the special case of uniform blades. Ground effects on helicopter rotors are included in the formulas.

The method of analysis, based on blade-element theory, is quite conventional, with the single exception of a semiempirical factor (k_1) introduced into Bernoulli's equation to take viscous effects into account. This factor affects the calculation of the induced velocity throughout the rotor disk, which is based in part on applying Bernoulli's equation between points representing the free stream, the top of the rotor, the bottom of the rotor, and the ground (in the case of ground effects).

Comparison is shown between theoretical calculations and experimental results. Satisfactory agreement is found. A curve of

k_t versus h/R (h = distance of rotor above ground, R = rotor radius) is given, based on experimental data.

Morris Morduchow, USA

Experimental Flow Equipment and Technique

(See also Revs. 948, 1002, 1015, 1031, 1059, 1060, 1062)

1044. **Pietro Teofilato, "Contribution to the analytic representation of a gas current by means of a hydraulic current (Contributo alla rappresentazione analitica di una corrente gassosa mediante una corrente idrica)," *Monogr. sci. Aero.*, Apr. 1947, no. 5, pp. 1–22.**

It has been proved by Riabouchinsky that a certain analogy exists between the flow of water and the flow of an imaginary gas whose ratio of the specific heats is equal to 2. This paper develops and establishes an analytical transformation which gives the relation between the hodograph of the flow of a real gas and that of the flow of the imaginary gas. This transformation permits the determination of the corresponding flow of water, or, if the results of experiments conducted in water channels are available, it permits analytical determination of the corresponding results for air. Thus some elaborate wind-tunnel experiments with air at high speeds may be replaced by simpler hydraulic tests.

The transformation mentioned above leads, in the general case, to rather involved expressions relating the velocity, pressure, and density of the air flow with the corresponding quantities of the water flow. Two important special cases are discussed, however, in which simple results are obtained. In the first of these the transformation is given only for points along a solid boundary rather than for the whole field of flow. In the second, the results are given for the complete field but are based on the assumption that the perturbations from an average velocity are small at all points. The results are applicable to subsonic as well as supersonic regimes.

Bruno A. Boley, USA

1045. **A. Thom and W. G. A. Perring, "The design and work of the Farnborough high-speed tunnel," *J. roy. aero. Soc.*, Apr. 1948, vol. 52, pp. 205–250.**

About one half of the article deals with the design, construction, and operation of the R.A.E. High-Speed Tunnel (working section 7 ft × 10 ft, pressure $^1/_{10}$–4 atm, power 4000 hp): description of the tunnel and the model tunnel ($^1/_{16}$ scale), and its cooling, air-drying, fan design, balances, speed measurements, leaks, and calibration. Some remarks about blockage corrections, strut corrections, and choking are made.

In the second half, results of measurements are discussed and a number of graphs are presented, including: (1) The influence of wing thickness, aspect ratio, and sweepback on the variation of the position of the aerodynamic center with Mach number; (2) pressure distribution and tuft observations on a (45-deg) swept-back wing of 14 per cent thickness; (3) measurements of the velocity distribution behind a Typhoon wing at tailplane position.

Three appendixes contain notes on estimation of tunnel power factor, cooling of the tunnel, and fan design.

E. Dobbinga, Holland

1046. **M. Tucker and M. D. Rousso, "Experimental investigation of the jet-boundary constriction correction for a model spanning a closed circular tunnel," *Nat. adv. Comm. Aero. tech. Note*, no. 1638, June 1948, pp. 1–30.**

An account is given of an experimental investigation of the low-speed tunnel wall corrections for a relatively large wing-fuselage model spanning a closed-throat circular wind tunnel. The measured variation of local constriction correction is shown as a function of the spanwise and chordwise location on the model. Average values of the constriction and induced curvature corrections are also presented.

Although the measured variation in local constriction correction was considerable, the average value for this correction was found to agree with the result of an approximate theoretical calculation. The experimental induced curvature correction was found to differ appreciably from the theoretical value. Because of the necessarily approximate nature of the theoretical calculations, the comparison between theory and experiment is not regarded by the authors as conclusive. The experimental results themselves, however, are of value in illustrating the effects of tunnel walls for a typical model of large size.

W. Vincenti, USA

1047. **E. Mostardi, "On the possibility of realization of high-speed wind tunnels utilizing boundary-layer suction on the diffuser walls (Sulla possibilità della realizzazione di gallerie aerodinamiche ad alta velocità con aspirazione dello strato limite dalla parete del diffusore)," *Aerotecnica*, Feb. 15, 1948, vol. 28, pp. 3–21.**

The purpose of this paper is to show potential saving in the power and over-all length of a wind tunnel by utilizing boundary-layer suction on the diffuser walls. First, estimates are made of the variation of energy ratio with area ratio, divergence angle, and over-all length for a series of conventional closed-throat, single-return wind tunnels, utilizing Wattendorf's formulas.

Experiments are then described which the author carried out in a small wind tunnel of the Istituto di Scienza of the University of Pisa concerning the influence of boundary-layer suction on diffuser efficiency. The experimental results give empirical factors which are used in conjunction with theoretical estimates of boundary-layer growth in diffusers, based on previous work of von Kármán, Margoulis, Gruschwitz, and Kehl.

Finally the foregoing theoretical and experimental work is applied to an illustrative wind-tunnel design. For this example the author estimates a potential gain in energy ratio from eight to sixteen by the application of boundary-layer suction, with a decrease of about 40 per cent in over-all tunnel length.

Frank L. Wattendorf, USA

1048. **D. D. Davis, Jr., and H. H. Sweberg, "Investigation of some factors affecting comparisons of wind-tunnel and flight measurements of maximum lift coefficients for a fighter-type airplane," *Nat. adv. Comm. Aero. tech. Note*, no. 1639, June 1948, pp. 1–32.**

Full-scale wind-tunnel tests were made on a single-engine fighter, to determine the effects of Reynolds number, propeller idling, and rate of change of angle of attack ($d\alpha/dt$) on maximum lift, flaps up and down. Values of $d\alpha/dt$ ranged from 0 to 85 deg per sec; Reynolds numbers were from 2.5×10^6 to 5×10^6. The effects of Reynolds number on this airplane were small. The effects of $d\alpha/dt$ were considerable, but smaller than were indicated by earlier measurements [Silverstein, Katzoff, and Hootman, *Nat. adv. Comm. Aero. Rep.*, no. 618, 1938] which covered only $^1/_{10}$ the range of the appropriate dimensionless parameter $(c/V)(d\alpha/dt)$. The results of tuft studies of the stall progression are included. Maximum lift coefficient decreased considerably as the advance ratio of the idling propeller increased from 0.7 to 1.4.

Comparable flight measurements were carried out on the same airplane. The comparison of results is quite satisfactory, providing that the wind-tunnel results are corrected rather elaborately.

W. R. Sears, USA

Hydraulics; Transport of Solids; Cavitation

(See also Revs. 972, 1059)

1049. Charles Jaeger, "Water-hammer effects in power conduits," *Civ. Engng. Lond.*, 1948, vol. 43, Feb., pp. 74–76; Mar., pp. 138–140; Apr., pp. 192–194; May, pp. 244–246.

This series of articles presents a collection of practical data relating to the design of hydroelectric penstocks and data regarding accidents and failures in operation. Nineteen cases of water-hammer effects are explained. Various types of surges and their causes are given. There is a discussion of auto-oscillation and the production of resonance by turbulence of flow, cavitation, and failure.

A plea is made for a frank exchange of experiences, since publication of details of past failures will help in analyzing and eliminating the causes of disturbances and possible accidents in service.

R. C. Binder, USA

1050. Ettore Scimemi, "On the validity of Thoma's rule for surge tanks at hydroelectric plants (Sulla validità della regola di Thoma per le vasche di oscillazione degli impianti idroelettrici)," *Energia Elett.*, Nov.-Dec. 1947, vol. 24, pp. 537–541.

D. Thoma has given (in 1910) the known formula for the smallest horizontal section acceptable for the chamber of an isolated working plant, with a long main conduit and a short pipe line. The author has made a study of several plants and found that in most cases the oscillations were damped, even for surge chamber sections notably smaller than Thoma's limit value. Undamped oscillations have been observed only at a single plant. Therefore the Thoma value should be considered overcautious and smaller values could be adopted.

Giulio De Marchi, Italy

1051. Augusto Ghetti, "Experimental investigations of the stability of governing of hydroelectric groups with pressure drop and surge tank (Ricerche sperimentali sulla stabilità di regolazione dei gruppi idroelettrici con derivazione in pressione e pozzo piezometrico)," *Energia Elett.*, Nov.-Dec. 1947, vol. 24, pp. 542–551.

An experimental water power plant has been set up at the hydraulic laboratory of the University of Padua for studying the stability by D. Thoma's method. To the 5.7-kw turbine with autonomous governor three cylindrical surge chambers with diameters from 0.15 to 0.30 m can be fitted: the maximum head is 8 m, the length of the penstock 48 m. The tests have shown that stability of governing was reached at surge chamber areas $^1/_3$ to $^1/_2$ of the Thoma value. This difference is explained by the failure of the governor to keep the power constant.

Giulio De Marchi, Italy

1052. Filippo Arredi, "The stability of adducing-generating systems of hydroelectric plants studied by the Leonhard criterion (Lo studio della stabilità dei sistemi adduttori-generatori degli impianti idroelettrici col criterio di Leonhard)," *Energia Elett.*, Nov.-Dec. 1947, vol. 24, pp. 559–566.

Let $H(p) = 0$ be the characteristic equation of an nth order linear equation with constant coefficients. Leonhard's theorem states that the solution is stable when in the $U + iV = H(u + iv)$ plane the transformed curve $H(iv)$ of the positive v-axis, directed from $v = 0$ to $v = \infty$, is a curve moving anticlockwise round the origin through n quadrants. The author shows a simple procedure of obtaining the expressions of U and V, from which the form of Leonhard's curve may be deduced by purely algebraic means. He applies the method to the study of the stability of hydroelectric plants with only a gallery, or with a gallery and surge tank, or with a gallery, surge tank, and conduit, and either for the case of constant hydraulic power, or for that of constant electric power.

Duilio Citrini, Italy

1053. Augusto Ghetti, "On the stability of oscillations in hydroelectric plants provided with a combined system of conduit and of surge tank (Sulla stabilità delle oscillazioni negli impianti idroelettrici provvisti di un sistema complesso di condotte e pozzi piezometrici)," *Energia Elett.*, Nov.-Dec. 1947, vol. 24, pp. 567–578.

Thoma's analysis of oscillations in the adducing systems of hydroelectric plants with surge tank, assuming constant power, is extended to complex situations with more than one gallery and more than one surge tank. The Hurwitz procedure is applied. The author, starting from a general case, deals then with some simpler situations, and particularly with a plant with only one gallery, but two surge tanks, one at the end and the other at an intermediate point of the gallery.

Duilio Citrini, Italy

1054. Charles Jaeger, "Steady flow in open channels: the problem of Boussinesq," *J. Instn. civ. Engrs.*, Feb. 1948, vol. 29, pp. 338–349.

This is a collection of previously published papers dealing with the "Bélanger-Böss generalized principle," according to which, for a given discharge, all flows in open channels, even nonlinear ones, can present a critical depth at which the energy head and the total impulse become a minimum. The author furnishes graphical representations of the relations between depth, discharge, energy head, and total impulse of a flow, and gives some applications of the principle (discontinuities of the bed or walls, hydraulic jump, erosion downstream of weirs, ground-water flow).

Duilio Citrini, Italy

1055. H. Holsters, "Calculation of the nonpermanent movement in rivers by the method called 'lines of influence' (Le calcul du mouvement non-permanent dans les rivières par la méthode dite des 'lignes d'influence')," *Rev. gén. Hyd.*, 1947, vol. 13, Jan.-Feb., pp. 36–39; Mar.-Apr., pp. 93–94; May-June, pp. 121–130; July-Aug., pp. 202–206; Sept.-Oct., pp. 237–245.

The method of "influence lines" is a method of graphical integration of the partial differential equations of flow, indicated by J. Massau in the *Annales de l'Association des Ingénieurs* between 1900 and 1905. Two equations, one the equation of motion and the other the equation of continuity, may be written involving depth and discharge as the unknowns. These quantities are unknown functions of time and distance along the channel. However, a pair of difference equations may be formulated which permit the determination of depth and discharge by successive approximations if the initial profile and two additional conditions are known.

The influence of factors such as variation in barometric pressure, wind, Chezy coefficient, irregularity of profile, and obstructions is discussed. Applications to the rivers of the Escant basin illustrate the method. Verification of the method by model tests of the Nèthe river system and by other experimental data indicates a high degree of precision for the procedure.

Glenn Murphy, USA

Marine Propulsion

(See also Rev. 942)

1056. S. B. Ralston, "A note on the block coefficient of a ship," *Trans. Instn. Engrs. Shipb. Scot.*, Dec. 1947, vol. 91, pp. 68–80.

Numerous empirical curves are employed by naval architects to determine the block coefficient for a ship of given length and speed. In this note a new family of curves for determining block coefficients, derived from ships built during recent years, is presented, in which the breadth of the ship is taken into account. Curves for the difference between block coefficient of after-body and that of the whole ship are also presented.

Louis Landweber, USA

1057. H. E. Saunders, "The multiple-skeg stern for ships," *Amer. Soc. nav. Archit. mar. Engrs.*, 1947, vol. 55, pp. 97–169.

The probable beneficial as well as detrimental effects resulting from the use of multiple skegs to improve the propulsive efficiency of multiple-screw ships are noted. Comparative test results are presented for conventional and twin-skeg designs of two types of twin-screw ships, a large tanker and a 700-ft passenger ship. In the twin-skeg design each propeller shaft is housed in a relatively fine skeg (deep keel) with a flat topped tunnel separating the skegs.

Although the resistance of both ships is higher with the twin-skeg stern, due to the added skin friction, the shaft horsepower, determined from self-propelled model tests, is 6 to 9 per cent less at the design speed. The change in stern of the passenger ship increased its propulsive coefficient from 0.66 to 0.73. Wake contours measured on the model of one of the twin-skeg designs are presented, and show considerable unsymmetry in the wake structure, thus indicating room for further improvement.

J. M. Robertson, USA

1058. J. A. Davies and H. W. Semar, "Marine engines—mechanical reduction gears," *Amer. Soc. nav. Archit. mar. Engrs.*, Nov. 1947, vol. 55, pp. 244–268.

This represents a review of current practice and trends in the design and performance of mechanical reduction gears for marine use. Gear-tooth types, methods of gear-tooth failure, gear-wheel materials and methods of construction, and the stresses and deflections in gear teeth are briefly discussed. Sources of gear-tooth errors are described, with methods of measuring such errors. Gear-tooth meshing losses are presented in relation to bearing and oil-spray losses, and it is suggested that the combined spray and mesh losses in modern speed reduction units may be taken as approximately 30 per cent of the bearing losses.

Milton C. Shaw, USA

1059. F. M. Lewis, "Propeller-tunnel notes," *Amer. Soc. nav. Archit. mar. Engrs.*, Nov. 1947, vol. 55, pp. 284–295.

The present procedure for cavitation tests on marine propellers in the Massachusetts Institute of Technology propeller tunnel is described, the tunnel itself having been described elsewhere. A comparison of open-tunnel, closed-tunnel, and open-water tests is made, and illustrated by data for a particular propeller.

It is pointed out that a closed tunnel is necessary for tests at small cavitation numbers (i.e., low pressure and high speed). At usual operating slip values, open-water and open-tunnel tests agree well, but not at very high and very low slips. Closed and open tunnel tests lead to differing results, which can apparently be reconciled by velocity corrections. Reynolds number, air content of the water, and irregularity of flow affect cavitation tests in tunnels; practical suggestions are given for reducing these troubles.

Finally, measured thrust and torque coefficients and cavitation conditions are shown in a set of graphs for a series of three related propellers.

A. O. Williams, Jr., USA

1060. H. F. Nordstrom, "Screw propeller characteristics" (in English), *Medd. SkeppsProvAnst. Goteborg*, no. 9, 1948, pp. 3–20.

The usual practice in testing marine propellers is to ascertain their characteristics when driving the ship in the forward direction, and very little data is available for any other condition. There are times however when it is important to know the characteristics of the propeller when reversing or when dragging, and in this case it is necessary to know the characteristics of the propeller in not only the forward rotation, forward speed quadrant, but for other combinations of speed and rotation as well.

This paper presents the results of a systematic set of propeller tests to ascertain the characteristics of a series of standard propellers of varying pitch ratio over a complete range of water speeds and propeller revolutions. A study of this nature has been needed for some time.

F. E. Reed, USA

1061. A. M. Robb, "An examination of the records of the Greyhound experiments," *Trans. Instn. nav. Archit. Lond.*, 1947, vol. 89, pp. 6–15.

The basic assumption for predicting ship resistance from tests of geometrically similar ship models may be stated as follows: The dimensionless resistance coefficient, $C_R = R/{}^1\!/_2\rho A v^2$, may be expressed as the sum $C_R = C_f + C_r$ where (1) C_f, the frictional resistance coefficient, is independent of ship form; (2) C_r, the residuary resistance coefficient, depends only upon the ship form and the Froude number $v/(gl)^{1/2}$.

The present paper principally questions the assumption that C_f is independent of the ship form. From a re-examination of the original experiments of William Froude it concludes that the assumption is seriously in error, and suggests that the frictional resistance of a ship form is appreciably less than that for the corresponding flat plate.

Considering that Froude's experiments were conducted more than 70 years ago, before the importance of surface roughness and of the nature of flow in the boundary layer was appreciated, the reviewer believes that definite conclusions cannot be derived from Froude's experiments. Both theory and more modern experiments, however, indicate that for sufficiently large ship models, Froude's assumption is a good approximation.

L. Landweber, USA

1062. F. H. Todd, "The determination of frictional resistance," *Shipbuilder*, Jan. 1947, vol. 54, pp. 15–19.

The author reviews the history and present status of methods of determining ship resistance. The method almost universally used was developed by William Froude in 1874. This method requires knowledge of the ship and model frictional resistances. To gain this knowledge, Froude towed thin planks edgewise through water. Similar tests were carried out later by others. R. E. Froude in 1887 reanalyzed his father's experiments, deduced an empirical formula, and extrapolated the results for ships of any size up to 1200 ft long. This formulation is still in use in England and permits fairly accurate prediction of ship resistance but does not consider Reynolds number.

In 1932 Schoenherr collected most of the then available results

and plotted them as ordinates of specific resistance against Reynolds number and proposed a new formulation (adopted in 1947 by the American Society of Naval Architects). Other critical studies of the problem have been made but no universally acceptable solution has been found. While a great deal is known of the problem, further experiments seem indicated. The most promising line of immediate progress seems to be carefully conducted trials with ships and a repetition of the "Greyhound" tests in which a full-sized ship is towed by another.

Karl E. Schoenherr, USA

Dynamics of Meteorology and Oceanography

1063. Robert G. Fleagle, "The fields of temperature, pressure, and three-dimensional motion in selected weather situations," *J. Met.*, Dec. 1947, vol. 4, pp. 165–185.

The author has obtained from data and calculations a composite picture of the temperature, pressure, and motion fields for three actual weather situations, one anticyclonic and two cyclonic systems. Emphasis was placed on the vertical velocities derived from aerological soundings.

The velocities show high correlations at some levels with the advection of temperature. The results shown indicate relations between the pressure, temperature, and motion fields, but do not explain or determine the physical principles involved.

Since the vertical velocity is derived from measurements of the other quantities of interest, there would seem to be some question as to whether the vertical velocity is an end product or a significant variable in the events examined. Philip Donely, USA

1064. W. V. R. Malkus, R. H. Bishop, and R. O. Briggs, "Analysis and preliminary design of an optical instrument for the measurement of drop size and free-water content of clouds," *Nat. adv. Comm. Aero. tech. Note*, no. 1622, June 1948, pp. 1–60.

A method for the determination of drop size and free water in clouds is described, and a mathematical analysis of rainbow theory is given in the appendix. The development of an optical instrument based on rainbow theory offers a more accurate and efficient method as compared with the usual manner of employing rotating cylinders, porous plugs, vaseline and soot slides, microphotographs, and differential dew points. The apparatus offers convenient operation by the observer, no disturbance of air sample, and ability to take readings as often as once a second. Further it is capable of being used in aircraft at temperatures both above and below freezing, and readily distinguishes between snow, sleet, or vapor.

The instrument is a preliminary design in that the water content and drop size must be kept constant during the interval of measurement. In addition to its uses in icing research in flight, the rainbow recorder should find application in the general study of clouds and fogs under conditions in which the instrument remains stationary. It may also be used in the study of particle size and distribution in carburetion problems dealing with liquid aerosols. S. K. Ghaswala, India

Ballistics; Detonics (Explosions)

(**See also Revs. 995, 1067**)

1065. W. Haack, "Projectile shapes for smallest wave drag," *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-3, 1948, pp. 1–53 (transl. from *Lilienthal Ges. Luftfahrtforsch. Ber.*, no. 139, part 1, 1941, pp. 14–37).

The paper gives computations of the meridional shape and wave drag of projectiles of minimum wave drag on the basis of the von Kármán-Moore linearized theory for supersonic flow about a slender body of revolution. The maximum cross-sectional area, length, and volume are assumed to be prescribed in the basic variational problem studied. Two of the quantities being chosen arbitrarily, the form with minimum wave drag is found by varying the third between its possible limits.

Considering a shape for which A is the largest cross-sectional area, V is the volume, and L the length, the results obtained are as follows: (1) A and V given, $L = (8/3)(V/A)$, $C_D = 16(4V^2/\pi L^4)$; (2) A and L given, $V = {}^1/_2 LA$, $C_D = (36/9)(4V^2/\pi L^4)$; (3) V and L given, $A = (16/9)(V/L)$, $C_D = (32/9)(4V^2/\pi L^4)$; (4) limiting nose with infinite radius of curvature at the rear, $A = (8/5)(V/L)$, $C_D = (96/25)(4V^2/\pi L^4)$. Comparison is made with experimental measurements of Ludwieg at Göttingen at a Mach number of two.

It is of interest that Zdĕnek Kopal has made a study of this paper and concludes that the shapes obtained by the author are not "slender" and exceed the range of useful applicability of the von Kármán-Moore theory. Hugh L. Dryden, USA

Thermodynamics

(**See also Revs. 1002, 1003, 1042, 1077**)

1066. A. Michels and S. R. De Groot, "Thermodynamical properties of carbon dioxide as function of density and temperature; of pressure and temperature" (in English), *Appl. sci. Res. Sec. A*, 1948, vol. 1, no. 2, pp. 94–102; pp. 103–106.

The specific heats, energy, enthalpy, entropy, free energy, thermodynamic potential, and kinetic energy of carbon dioxide have been recalculated for pressures up to 3000 atm between 25 and 125 C. Tables of these properties as a function of density and temperature are given.

The major change from previous calculations has been in the specific heats at zero pressure. These have now been calculated from the four fundamental frequencies of the CO_2 molecule based on spectroscopic data. The formulas used in obtaining the various thermodynamic properties are also included.

The second paper presents tables for the pressure-volume product, enthalpy, energy, entropy, free energy, thermodynamic potential, and internal energy of carbon dioxide for pressures up to 2000 atm between 25 and 150 C.

The two papers represent an important contribution to knowledge of the thermodynamic properties of carbon dioxide over a wide range of pressure and a limited range of temperature.

Joseph Kaye, USA

1067. Bernard Lewis and Guenther von Elbe, "Ignition and flame stabilization in gases," *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 307–316.

The authors summarize their own work and that of others. Self-ignition (the transition from slow to very rapid chemical reaction with approximately adiabatic temperature rise) occurs through thermal or branched-chain explosion. In the former, the chemical reaction becomes self-accelerating owing to the temperature rise. In the latter, the self-acceleration is caused by the formation of chemically active particles at a rate that exceeds the rate of destruction of these particles. Local ignition, as by a spark, results in the formation of a combustion wave propagated according to Huyghens' principle.

The minimum ignition requirement of electric capacitance sparks in mixtures of methane or other hydrocarbons with oxygen

and nitrogen bears only on the spark energy; the minimum spark energy is shown to be a function of the burning velocity, the width of the combustion wave, and other variables of the gas mixture, in agreement with experimental data. A combustion wave in a stream becomes stationary when the burning velocity is equal to the gas velocity somewhere in the wave, but nowhere exceeds it. The mechanism of equalization of burning velocity and gas velocity at the rim of a burner tube is described. It is shown that the limits for flashback and blowoff correspond to critical values of the gas-velocity gradient at the stream boundary, inner or outer.

Stuart R. Brinkley, Jr., USA

1068. **J. L. Finck, "Thermodynamics, Part II: Work, heat, and temperature concepts, and an examination of the temperature scale,"** *J. Franklin Inst.*, May 1948, vol. 245, pp. 365–378.

The concepts of work, heat, and temperature are examined from the standpoint of a complete equation for the internal energy of a system. The concepts presented in Part I [*J. Franklin Inst.*, Mar. 1948, vol. 245, p. 301] are utilized. Since the Kelvin temperature scale is designed for a perfect gas, it is based on an equation for the internal energy which is incomplete for any practical system. A temperature scale based on a complete equation is shown to have certain theoretical advantages over the Kelvin scale. It is shown that the temperature of the absolute zero as based on a complete equation is lower than that on the Kelvin scale.

A suggestion is offered whereby an actual system may be treated as complete, and a temperature scale may be based on this. Using published data on several systems which may be treated as complete, values of temperature are derived, and these values show considerable deviation. More exact experimental data are therefore required to check the feasibility of such a temperature scale.

J. Howard Childs, USA

1069. **Carl A. Zapffe, "Concept of the hydrogen potential in steam-metal reactions,"** *Trans. Amer. Soc. Metals*, 1948, vol. 40, pp. 315–354.

As the author states, he makes a thermodynamic study of numerous reactions of metals and alloys with moisture, and develops the concept of the "hydrogen potential"—a chemical attribute of the hydrogen released during the moment of reaction, which generates known or calculable pressures of hydrogen in a closed system and which can therefore in turn be measured for dynamic systems in terms of those pressures.

Since the absorption of hydrogen by metals is a function of the hydrogen pressure, these "hydrogen potentials" provide a basis for calculating the liability of the various metal systems to hydrogen pickup from the moisture reaction.

Calculations for several metals are presented in graph form, and the significance of the plots is discussed. Conditions over a wide range of temperature and a wide range of humidities and steam pressures yield calculations which readily explain numerous metallurgical phenomena.

J. Howard Childs, USA

1070. **J. J. Hermans, "Diffusion with discontinuous boundary,"** *J. Colloid Sci.*, Aug. 1947, vol. 2, pp. 387–398.

If the molecules of a diffusing substance react chemically with the medium into which the substance is diffusing, they become fixed in position or are removed from the diffusion process. The result is that a sharp boundary line is established which changes in position with time, but remains perfectly sharp throughout the process. Three simple cases of this sort are here analyzed mathematically and discussed in detail.

Some preliminary experiments, performed to test the above theory, checked the theory within experimental error in one case, while in a second case the agreement was poor. Further experimental work is in progress.

Joseph Kaye, USA

1071. **J. E. Verschaffelt, "On the thermomechanics of open systems (Sur la thermomécanique des systèmes ouverts),"** *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 1, pp. 17–38.

The author establishes the fundamental thermodynamic laws for systems with variable mass by adding supplementary terms. The resulting equations are illustrated by means of a series of examples.

The generalized equations are primarily of theoretical interest. For complicated chemico-physical processes or biological applications it may be of interest to use the equations for open systems.

E. Haenni, USA

1072. **J. V. Becker and D. D. Baals, "Analysis of heat and compressibility effects in internal flow systems and high-speed tests of a ramjet system,"** *Nat. adv. Comm. Aero. Rep.*, no. 773, 1943 (issued in 1948), pp. 1–40.

This report is a thorough analysis of internal flow systems. It accounts for the effects of heating and compressibility, and incorporates the findings of some previous reports on this subject. Equations and charts are presented whereby the flow characteristics at important stations of the system may be readily obtained. This paper is of great value for the analysis of duct work on all applications, particularly where high internal Mach numbers and heat input are of importance.

A number of tests are reported which show the validity of the equations, which are based on one-dimensional flow theory and uniform velocity distribution. The theory is expanded to cover the ideal ramjet engine. While the results of this analysis are interesting and correct for the assumptions made, recent literature on the theory of ramjets gives more complete information.

H. E. Sheets, USA

1073. **S. R. De Groot and A. Michels, "The Joule-Thomson effect and the specific heat at constant pressure of carbon dioxide"** (in English), *Physica Hague*, May 1948, vol. 14, pp. 218–222.

The authors calculate the Joule-Thomson coefficient μ of CO_2 from measurements of enthalpy and density. For the range 0 to 2000 atm and 25 to 150 C, the specific heat c_p and μ are given, as well as the inversion curve of the Joule-Thomson effect. The calculations employ an uncited empirical formula for specific volume. Where the experimental data by Roebuck, Murrell, and Miller are available, reasonable agreement between observed and calculated data of μ exists.

E. F. Lype, USA

Heat Transfer

(See also Revs. 995, 1026)

1074. **H. Julian Allen and Bonne C. Look, "A method for calculating heat transfer in the laminar flow region of bodies,"** *Nat. adv. Comm. Aero. Rep.*, no. 764, 1943 (issued in 1948), pp. 1–6.

In this paper the authors apply the Reynolds analogy to the calculation of heat transfer in the laminar flow regions of bodies.

They assume that this analogy is applicable in this region provided only that the Prandtl number is unity. No justification is

given for the neglect of the other conditions of applicability for this analogy; namely, that the pressure gradient $\partial p/\partial x$ must be zero, and that the boundary conditions for the temperature and velocity are analogous [compare Goldstein, "Modern developments in fluid dynamics," Oxford Univ. Press, 1938, vol. 2, p. 651]. This leads to some erroneous conclusions, such as that the heat transfer at the laminar separation point is zero. One sample calculation, however, indicates that the average heat transfer thus computed is in fair agreement with experiment.

J. M. Wild, USA

1075. M. Michaud, "Emissivity of cavities of simple geometric forms (Facteur d'émission des cavités de formes géométriques simples)," *C. R. Acad. Sci. Paris*, Mar 22, 1948, vol. 226, pp. 999–1000.

A method of modification is presented for use with the classical formulas for predicting the effective emissivities of imperfect black bodies, and for use with formulas which take into account multiple reflections within cavities. In effect, the technique consists of substituting plane angle relationships for the solid angle relationships in the classical formulas. Experimental values of effective emissivities are compared with values calculated from the modified formulas. Good agreements are achieved.

Kenneth R. Wadleigh, USA

1076. I. A. Keebel, "Heating of a viscous fluid by a rotating disk" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Nov.-Dec. 1947, vol. 11, pp. 611–614.

Von Kármán in 1921 and Cochren in 1934 showed that the equations of steady motion of incompressible viscous fluid have exact solutions of the form

$$v_r = rf(z);\ v_\theta = rg(z);\ v_z = h(z);\ p = p(z) \qquad [1]$$

(v_r, v_θ, v_z being velocity components in cylindrical co-ordinates) with the condition that f, g, h, and p satisfy a given system of four differential equations of second degree. This motion is ordinarily interpreted as due to the rotation of a flat plate of great radius $z = 0$ around the z-axis. There are boundary conditions involving the angular velocity ω and the pressure p_o on the surface of the disk. The inverse problem, of a fluid which is rotating and is slowed by a stationary plate $z = 0$ immersed in it, was treated by Bödewadt in 1940.

The author treats the problem of the temperature distribution for both of the above cases. The equation of heat flow is established for steady motion symmetrical about the axis z in the form

$$c\rho\left(v_r\frac{\partial T}{\partial r} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right]$$
$$+ A\mu\left[2\left(\frac{\partial v_r}{\partial r}\right)^2 + 2\left(\frac{v_r}{r}\right)^2 + 2\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)^2 + \left(\frac{\partial v_\theta}{\partial z}\right)^2 + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)^2\right]$$

T being temperature, c specific heat, k thermal conductivity, and A thermal equivalent.

This equation is shown to be satisfied if the velocities are found according to Equations [1], and the temperature is expressed as $T = T_1(z) + T_2(z)\,r^2$. Afterwards, for the definition of the functions of [1], the author, like von Kármán and Cochren, introduces the dimensionless coefficients F, G, H, ζ in the form

$$f = \omega F;\ g = \omega G;\ h = \sqrt{\nu\omega}H;\ z = \sqrt{\nu/\omega}\,\zeta$$

T_1 and T_2 are replaced by the functions τ_1 and τ_2 given by

$$T_1(z) = A\,\frac{\mu\omega\nu}{k}\,\tau_1(\zeta);\ T_2(z) = A\,\frac{\mu\omega^2}{k}\,\tau_2(\zeta)$$

For the integration H, G, F are assumed constant. Two cases are considered, when the Prandtl number is and is not unity. The limits are calculated for two assumptions: (1) $\partial T/\partial z = 0$; and (2) $T_o = C^{te}$. The result of the integration is used for finding numerical values of τ_1 and τ_2, which are tabulated.

D. Jacovleff, Belgium

1077. E. E. Callaghan and R. S. Ruggeri, "Investigation of the penetration of an air jet directed perpendicularly to an air stream," *Nat. adv. Comm. Aero. tech. Note*, no. 1615, June 1948, pp. 1–13.

An experimental investigation is described on the penetration of hot air jets (400 F) perpendicularly into an air stream of 260 to 360 fps velocity in a 2 × 20-in-duct wind tunnel. The jets were discharged from $^1/_4$ to $^5/_8$-in-diam thin plate orifices at 5 to 50 in. Hg pressure drop across the orifice. Results are plotted in dimensionless form (penetration coefficient versus a composite parameter), and a simple empirical equation is developed for the depth of penetration as a function of downstream distance, orifice diameter, and the relative velocities and densities. The data are of interest in connection with the heating of air streams by mixing with hot or burning gases, where a high degree of turbulence and mixing is desirable.

Otmar E. Teichmann, USA